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DOUBLE-CANTILEVER BEAM SPECIMEN BENT BY PAIRS OF OPPOSITE TERMINAL TRANSVERSE LOADS

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ABSTRACT

A double-cantilever beam (*DCB*) specimen embedded horizontally in a wall at one end and bent by pairs of opposite terminal transverse loads ***F*** and ***-F*** at the other end presents two different behaviors: (*i*) When *F* is small, the separation of the two beams would end before reaching the wall. (*ii*) For *F* sufficiently large, a crack would develop and expand towards the wall. Assuming a constant crack-front velocity *v*, the crack extension force *G* per unit length of the crack-front can be calculated. Its value $G^{(I)}$ is provided in the subsonic velocity regime ($v < c_t$ velocity of transverse sound wave), as example.

Keywords : *fracture mechanics, linear elasticity, dislocations, crack extension force, mechanical behavior of materials.*

RÉSUMÉ

Échantillon de poutre à double porte-à-faux plié par des paires de charges transversales terminales opposées

Un échantillon de poutre à double porte-à-faux (*DCB*) encastré horizontalement dans un mur à une extrémité et plié par des paires de charges transversales terminales opposées ***F*** et ***-F*** à l'autre extrémité présente deux comportements différents : (*i*) Lorsque *F* est petit, la séparation des deux poutres se terminerait avant d'atteindre le mur. (*ii*) Pour *F* suffisamment grand, une fissure se développerait et s'étendrait vers le mur. En supposant une vitesse de front de fissure constante *v*, la force d'extension de fissure *G* par unité de longueur du front de fissure peut être calculée. Sa valeur $G^{(I)}$ est fournie dans le régime de vitesse subsonique ($v < c_t$, la célérité des ondes transversales), à titre d'exemple.

Mots-clés : *mécanique de la rupture, élasticité linéaire, dislocation, force d'extension de fissure, propriétés mécaniques des matériaux.*

Consider a rectangular bar *BAA'B'* **(*Figure 1*)** of thickness *BB'* = 2 e_0, embedded in wall at one end *BB'* in a horizontal position; at the other end *AA'*, a pair of opposite terminal transverse vertical loads ***F*** and ***-F*** separate into two equal parts (*J*) BAA_mB_m and (*J*)' $B_mA_mA'B'$ the bar *BAA'B'*. These are drawn dashed before separation and full under loads. Such specimen under loads is named " Double-cantilever beam (*DCB*) specimen" (see Fig. 2.13 in [1]) with beams (*J*) and (*J*)'. To the initial dashed form of the specimen is attached a Cartesian coordinate system x_i where the direction x_1 (left-to-right) is horizontal (axis of the sample), x_2 is the upward vertical and $x_1x_2x_3$ forms a direct orthogonal basis. An origin for this coordinate system may be located at the center of the sample cross section at the wall.

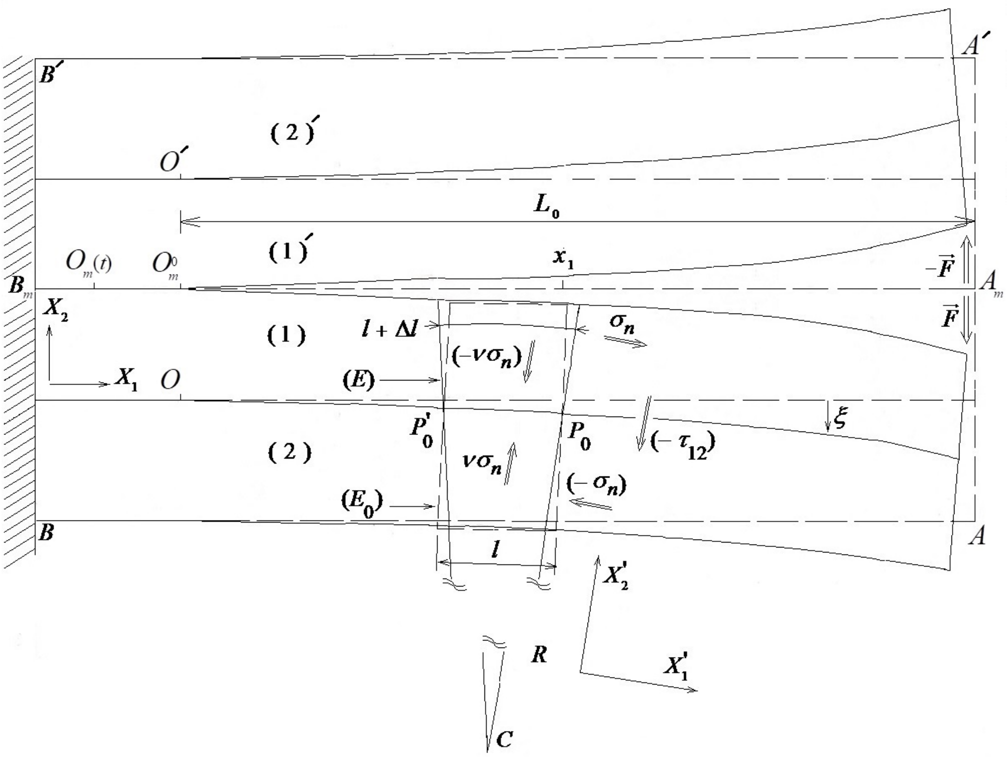

Figure 1 : *Rectangular bar BAA'B'* (*length BA, thickness BB'* = 2 e_0 *and width* l_0, *measured along* x_1, x_2 *and* x_3, *respectively*) *embedded in a wall at one end BB' in horizontal position and separated into two arms at the other end AA' by a pair of opposite vertical loads* ***F*** *and* ***-F***. *It is assumed that no shearing stresses exist in the sample region located between vertical lines* BB_mB' *and* OO_m^0O'; *see text for details.*

The objective of this study is to investigate the conditions for the separation of the bar under loads into to equal parts (*J*) and (*J*)'. The specimen is assumed large, homogeneous, isotropic, deforming elastically, and fracturing without plasticity. Gravitational forces are neglected. The elastic stress fields in the *DCB* sample can be understood from [2]. The bottom part (*J*) of the specimen can be viewed as a rectangular bar bent by terminal transverse load ***F***. Hence, various notations and definitions reported in part (*J*) ***(Figure 1)*** are identical to those of Fig. 1 of [2]; remains to specify what is L_0 in the present. It is assumed that to given F corresponds a position O_m^0 such that $O_m^0 A_m = L_0$. Moving on the central line from A_m to B_m, O_m^0 is the first spatial position for which the two arms (*J*) and (*J*)' have horizontal tangents parallel to $B_m A_m$ (*i.e.* to x_1). Under such conditions, the tensile stresses $\sigma_{11} \equiv \sigma_n(O_m^0)$ at O_m^0 in both (*J*) and (*J*)' are parallel to x_1, and read [2] :

$$\sigma_n(O_m^0) = \frac{F L_0 e_0}{2I}, \qquad (1)$$

$$I = \frac{l_0 e_0^3}{12}.$$

Considering tensile forces $\sigma_n(O_m^0)$ only, there is no relative displacement of faces of the crack in the x_1 and x_2 directions. Consequently, the crack would stop about O_m^0; this corresponds to $G(O_m^0) = 0$. However there exists tensile stresses parallel to x_2 [2] that open the crack faces about O_m^0. These are $\sigma_{22}(O_m^0)' \equiv \nu \sigma_n(O_m^0)$ and $\sigma_{22}(O_m^0) \equiv -\nu \sigma_n(O_m^0)$ in arms (*J*)' and (*J*) (ν is Poisson's modulus). In x_1- region between B_m and O_m^0, the tip of the crack is under mode *I* loading. Assuming a planar straight-fronted crack in uniform motion v = constant, the crack extension force G can be taken from [3]. Hence in the subsonic velocity regime ($v < c_t$ the velocity of transverse sound wave), we may write:

$$G^{(I)}\left(O_m(t)\right) = \frac{b K_I^2}{4\pi C_1^{(I)}}, \qquad (2)$$

$$K_I = \nu \sigma_n(O_m^0)\sqrt{\pi c}, \; c = vt + L_0,$$

$$C_1^{(I)} = \frac{\mu b\left(2 - \tilde{v}_t^2\right)}{\pi \tilde{v}_t^2}\left(\frac{1}{P_t} - \frac{1}{P_l}\right);$$

$$\tilde{v}_t = v / c_t, \quad P_t^2 = 1 - \tilde{v}_t^2; \quad \tilde{v}_l = v / c_l, \quad P_l^2 = 1 - \tilde{v}_l^2;$$

μ is the shear modulus, c_l the velocity of longitudinal sound wave, t incremental time from zero and b a Burgers vector used in [3]. It is stressed that O_m^0 position along B_mA_m ***(Figure 1)*** may be determined experimentally by observing the shape of the bar *BAA'B'* under loads; *BB'* is allowed locally to contract and extend freely along x_2 in the wall. In summary, considering a *DCB* specimen bent by pairs of opposite terminal transverse loads ***F*** and ***-F (Figure 1)***, two different situations emerge: (*i*) When F is small, the separation into two beams would end about O_m^0 before reaching the wall; this corresponds to $G(O_m^0) = 0$. (*ii*) For sufficiently large F, a crack would develop from O_m^0 towards B_m. Assuming its motion uniform with constant crack-front velocity v, the crack extension force G per unit length of the crack-front is finite. Its value $G^{(I)}$ is given by (2). We may therefore equal $G^{(I)}$ to 2γ where γ is the surface energy under stationary conditions.

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