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NON-PLANAR INTERFACE CRACK UNDER GENERAL LOADING II. *CLIMB-TYPE SINUSOIDAL EDGE DISLOCATION*

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ABSTRACT

This study's objective is to analyse the conditions of propagation of an oscillatory front crack along a non-planar interface, under mixed mode I + II + III loading. The crack model consists of a continuous distribution of three families of non-straight dislocations having the shape of the crack front : families 1 and 2 are edges (on average) and family 3 is screw. The associated Burgers vectors $\vec{b}_j$ (j=I, II, III) are directed along the applied tension and shears x_2, x_1 and x_3 directions, respectively. The dislocations are aligned along the x_3 – direction and spread in x_2x_3 – planes in a small oscillating form $\xi(x_1, x_3)$ at an average elevation $h(x_1)$. It is sufficient to investigate a simpler model where the interface is in the form of a corrugated sheet and the dislocations have a sinusoidal form. The first part I of this study has provided expressions for the displacement and stress fields of sinusoidal dislocations with $\vec{b}_I$ (glide-type edges) and $\vec{b}_{III}$ (screws). Those $\vec{u}^{(m)}$ and $(\sigma)^{(m)}$ of dislocation family 2 with $\vec{b}_{II}$ (climb-type edges) are the subject of this part II. It is shown that in order to guarantee conditions of continuity of the elastic fields at the crossing of the interface, the relative variation of volume must be continuous at the crossing of the interface with well-defined non-zero values which depend on Poisson's ratios. This results in an estimated mismatch, far from the interface, between the displacements $\vec{u}^{(m)}$ and $\vec{u}^{(m)\infty}$; the latter corresponding to the field due to the same dislocation in a totally homogeneous medium.

Keywords : *linear elasticity, interface dislocations, Galerkin vector, three-dimensional biharmonic functions, Fourier forms, linear systems of equations.*

RÉSUMÉ

Fissure d'interface non plane sous sollicitation extérieure arbitraire II. *Dislocation coin de type montée*

La présente étude se fixe pour objectif d'analyser les conditions de propagation d'une fissure de front oscillatoire le long d'une interface non plane sous sollicitation en mode mixte I+II+III. Le modèle de fissure adopté, est une distribution continue de trois familles de dislocations non rectilignes ayant la forme du front de fissure : les familles 1 et 2 sont des coins (en moyenne) et la famille 3 est vis. Les vecteurs de Burgers infinitésimaux associés $\vec{b}_j$ (j= I, II, III) sont suivant les directions x_2, x_1 et x_3, correspondant à la tension et aux cisaillements appliqués, respectivement. Les dislocations sont suivant la direction x_3 et s'étalent dans les plans x_2x_3 dans la forme $\xi(x_1, x_3)$ à la hauteur $h(x_1)$. Il suffit d'analyser un modèle de fissure plus simple dans lequel l'interface a la forme d'une tôle ondulée et les dislocations sont sinusoïdales. La première partie de cette étude a fourni les expressions des champs de déplacement et contrainte des dislocations sinusoïdales avec $\vec{b}_I$ (coins de type "glissile") et $\vec{b}_{III}$ (vis). Ceux de la famille 2 avec $\vec{b}_{II}$ (coins de type "montée") sont l'objet de cette partie II. On montre que, dans le but de garantir la continuité des champs élastiques à la traversée de l'interface, la variation relative de volume doit être continue au passage de l'interface avec une valeur bien définie, non nulle, laquelle dépend des rapports de Poisson des milieux environnants. Il en résulte un écart estimé, loin de l'interface, entre les déplacements $\vec{u}^{(m)}$ et $\vec{u}^{(m)\infty}$; le dernier correspondant aux champs de la même dislocation dans un milieu totalement homogène.

Mots-clés : *élasticité linéaire, dislocations d'interface, Vecteur de Galerkin, fonctions biharmoniques à trois dimensions, expansions en séries de Fourier, systèmes d'équations linéaires.*

I - INTRODUCTION

This study is involved with the determination of the elastic fields of a non-planar interface crack model in equilibrium under an externally applied general loading. The model of crack and its pertinence to represent large real cracks have been introduced in part I of this work [1] ; it consists of two infinitely extended elastic mediums *R*1 and *R*2 firmly welded along a non-planar interface *R* ***(Figure 1)***. With respect to a Cartesian coordinate system x_i, the crack extends from $x_1 = -a$ to *a*, fluctuates on average about Ox_1x_3-plane; its front runs indefinitely in the x_3- direction and spreads in x_2x_3 – planes in the Fourier series form *f*, written as

$$f = \sum_n \left(\xi_n \sin \kappa_n x_3 + \delta_n \cos \kappa_n x_3\right) + h \equiv \xi + h \quad (1)$$

where *n* is a positive integer; *h*, ξ_n, δ_n and κ_n are real numbers that depend on position x_1 along the crack length. The system is subjected to remote applied

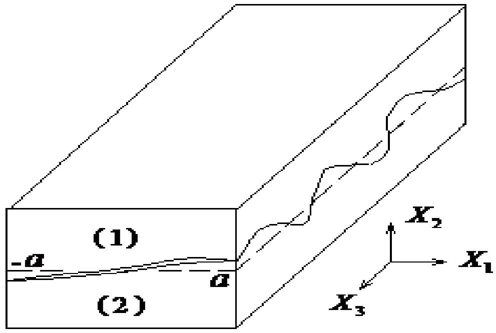

Figure 1 : *Schematic illustration of the crack front in two elastic solids (1) and (2) welded along a non-planar wavy surface that contains an interface crack. The crack fronts lie in* $x_2 x_3$ *– planes in the form* f *(1); in this geometry, the system is subjected to mixed mode I+II+III loading with the applied tension in the* x_2 *– direction. The average fracture surface (dashed) is shown perpendicular to that direction*

general loading, corresponding to a tension along x_2, and two shears (parallel to $x_1 x_3$) directed along x_1 and x_3, respectively. Our method of analysis consists in representing the crack by a continuous distribution of three families of non-straight infinitesimal dislocations having the form *f*. Families 1 and 2 are edges (on average) and family 3 is screw. The associated Burgers vectors $\vec{b}_j$ (*j*=I, II, III) are directed along the applied tension and shears x_2, x_1 and x_3 directions, respectively. It is sufficient to investigate a simpler model where the interface is in the form of a corrugated surface *S* and the dislocations have a sinusoidal form A_n given by

$$A_n = \xi_n \sin \kappa_n x_3 \, . \quad (2)$$

The applied loadings and dislocation Burgers vectors are unchanged. The case of a general shape, with crack front *f* (1), is achieved by superposition [1]. From the elastic fields of the dislocations, the crack-tip stress and crack extension force can be evaluated. Such analyses, with variable complexity of the crack front, exist in the case of an infinitely extended isotropic medium ([2 - 9], among others). We plan to extend the analysis to non-planar interface cracks under general loading (mixed mode I+II+III). The first part I of this study [1] has provided expressions for the displacement and stress fields of sinusoidal dislocations with $\vec{b}_I$ (glide-type edges) and $\vec{b}_{III}$ (screws). Those of dislocation family 2 with $\vec{b}_{II}$ are the subject of this part II. For the various types of dislocation, the general geometry is that of ***Figure 2***; because $\vec{b}_{II} = (b,0,0)$ is out of the Ox_2x_3 –plane of location of the dislocation, this is a climb-type edge. In what follows, the methodology and elastic field expressions form Section 2 and 3, respectively. Discussion and conclusion form Section 4 and 5 respectively. The third part III of the work will deal with crack-tip stresses and crack extension force when the non-planar interface crack is loaded in mixed mode I+II+III.

II - METHODOLOGY

We seek the elastic fields of a dislocation ($\vec{b}_{II} = (b,0,0)$) lying on a non-planar interface S having the form of a corrugated sheet that separates two firmly welded elastic solids $S1$ and $S2$ of infinite sizes. S is defined by the point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$ and $S1$ and $S2$ occupy the regions $x_2 > \xi_n \sin \kappa_n x_3$ and $x_2 < \xi_n \sin \kappa_n x_3$, respectively. The situation is shown in ***Figure 2*** where $S1$ and $S2$ are confined for illustration purpose in a parallelepiped of finite sizes. The dislocation is located at the origin, runs indefinitely in the x_3 – direction and spreads in the x_2x_3 – plane in the form A_n (2).

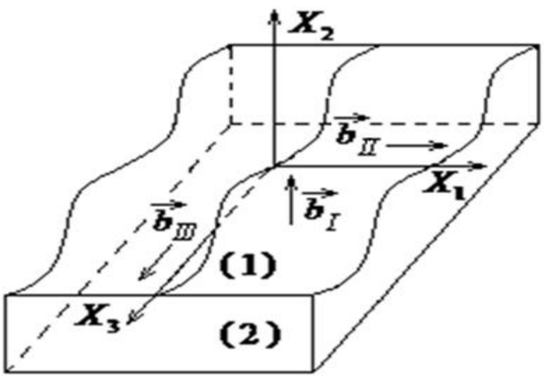

Figure 2 : *Two elastic mediums (1) and (2) welded along a non-planar sinusoidal surface and containing an interface sinusoidal dislocation at the origin. The dislocation lies in the* Ox_2x_3 – *plane in the form* A_n *and runs indefinitely in the* x_3 – *direction*

The methodology has been detailed in different places [1, 10, 11]. The elastic fields $(\vec{u}^{(m)}, (\sigma)^{(m)})$ are assumed to be the difference between two quantities $(\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty})$ and $(\vec{u}^{(m)W}, (\sigma)^{(m)W})$:

$$\begin{aligned} \vec{u}^{(m)} &= \vec{u}^{(m)\infty} - \vec{u}^{(m)W} \\ (\sigma)^{(m)} &= (\sigma)^{(m)\infty} - (\sigma)^{(m)W} \end{aligned} . \tag{3}$$

The former with ∞ corresponds to the fields of a sinusoidal dislocation (edge or screw) in an infinitely extended homogeneous solid (*m*); the latter with *W* satisfies the equations of equilibrium and is constructed in such a way that : (*a*) $(\vec{u}^{(m)}, (\sigma)^{(m)})$ are continuous at the crossing of the interface, implying that

$$\begin{aligned} \Delta\vec{u}^{\infty}(P_S) &\equiv \vec{u}^{(2)\infty} - \vec{u}^{(1)\infty} = \vec{u}^{(2)W} - \vec{u}^{(1)W} \equiv \Delta\vec{u}^{W}(P_S) \\ (\Delta\sigma)^{\infty}(P_S) &\equiv (\sigma)^{(2)\infty} - (\sigma)^{(1)\infty} = (\sigma)^{(2)W} - (\sigma)^{(1)W} \equiv (\Delta\sigma)^{W}(P_S)\,; \end{aligned} \tag{4}$$

(*b*) $(\vec{u}^{(m)}, (\sigma)^{(m)})$ tends to $(\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty})$ when one moves far away from the interface in the x_2 – direction. This means that

$$\begin{aligned} \vec{u}^{(m)W} &\to 0 \\ (\sigma)^{(m)W} &\to 0 \end{aligned} \tag{5}$$

when $|x_2| \to \infty$. $(\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty})$ may be taken from [6, 7]; they are given to linear expressions with respect to ξ_n . The associated terms $(\vec{u}^{(0)(m)\infty}, (\sigma)^{(0)(m)\infty})$ of zero order correspond to the fields of a straight dislocation and second terms $(\vec{u}^{A_n(m)\infty}$, $(\sigma)^{A_n(m)\infty})$ are proportional to either A_n or its spatial derivative $\partial A_n / x_3$. Hence, we have

$$\begin{aligned} \Delta\vec{u}^{\infty}(P_S) &= \Delta\vec{u}^{(0)\infty} + \Delta\vec{u}^{A_n\infty} \\ (\Delta\sigma)^{\infty}(P_S) &= (\Delta\sigma)^{(0)\infty} + (\Delta\sigma)^{A_n\infty} \end{aligned} \tag{6}$$

on the interface point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$; Appendix A below gives the complete list, component by component, for the climb-type sinusoidal edge dislocation; the corresponding values for the glide-type edge and screw sinusoidal dislocation have been given in [1, 10, 11]. $(\vec{u}^{(m)W}, (\sigma)^{(m)W})$ are obtained with the help of Galerkin vectors; these are available for the glide-type edge and screw (see [1, 10, 11]). For the climb-type sinusoidal edge, we arrive at Galerkin vectors $\vec{V}$ with only one non-zero x_1 – component, arranged in the form

$$V_1(\vec{x}) = \overline{\alpha_1}(\vec{k})e^{i\vec{k}.\vec{x}} + \overline{\beta_1}(\vec{k})x_2 e^{i\vec{k}.\vec{x}} \tag{7}$$

under the condition $\vec{k}^2 = k_1^2 + k_2^2 + k_3^2 = 0$ that ensures the biharmonicity of V_1. For V_1 to cancel far from the interface, we write

$$k_2 = k_2^{(m)} \equiv (-1)^{m-1} i \sqrt{k_1^2 + k_3^2} \quad (8)$$

with $m = 1$ when $x_2 > \xi_n \sin \kappa_n x_3$ (half-space 1) and $m = 2$ when $x_2 < \xi_n \sin \kappa_n x_3$ (half-space 2). We use the notations

$$\vec{k}^{(m)} \equiv (k_1, k_2^{(m)}, k_3)\,,\ \overline{\alpha}_1^{(m)} \equiv \overline{\alpha}_1(\vec{k}^{(m)})\,,\ \overline{\beta}_1^{(m)} \equiv \overline{\beta}_1(\vec{k}^{(m)})\,;$$

hence for half-space 1 ($x_2 > \xi_n \sin \kappa_n x_3$), solid (1)

$$V_1(\vec{x}) \equiv V_1^{(1)}(\vec{x}) = \overline{\alpha}_1^{(1)} e^{i\vec{k}^{(1)}.\vec{x}} + \overline{\beta}_1^{(1)} x_2 e^{i\vec{k}^{(1)}.\vec{x}}$$

and for half-space 2 ($x_2 < \xi_n \sin \kappa_n x_3$), solid (2)

$$V_1(\vec{x}) \equiv V_1^{(2)}(\vec{x}) = \overline{\alpha}_1^{(2)} e^{i\vec{k}^{(2)}.\vec{x}} + \overline{\beta}_1^{(2)} x_2 e^{i\vec{k}^{(2)}.\vec{x}}\,.$$

The elastic fields corresponding to V_1 (7) may be first calculated; then, more general forms $\vec{u}^{(m)V}$ and $(\sigma)^{(m)V}$ are constructed from the previous ones by superposition over k_1 and k_3; we may write

$$\begin{aligned} \vec{u}^{(m)V} &= \vec{u}^{(m)A} + \vec{u}^{(m)B} = \vec{u}^{(m)A+B} \\ (\sigma)^{(m)V} &= (\sigma)^{(m)A} + (\sigma)^{(m)B} = (\sigma)^{(m)A+B} \end{aligned} \quad (9)$$

where terms with *A* and *B* refer to $\overline{\alpha}_1$ and $\overline{\beta}_1$ in (7) respectively. For $\vec{u}^{(m)V}$ and $(\sigma)^{(m)V}$ to conform with $\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$, the summation over k_1 is continuous and that over k_3 is discrete. k_3 takes three values: $-\kappa_n$, 0, κ_n. The fields corresponding to $k_3 = 0$ are denoted $\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$, and terms associated with $k_3 = -\kappa_n$ and κ_n are merged to form expressions denoted by $\vec{u}^{A_n(m)V}$ and $(\sigma)^{A_n(m)V}$; this is made possible by requiring that

$$\overline{\alpha}_1^{(m)}(\kappa_n) \equiv -\overline{\alpha}_1^{(m)}(-\kappa_n)\,,\ \overline{\beta}_1^{(m)}(\kappa_n) \equiv -\overline{\beta}_1^{(m)}(-\kappa_n)\,. \quad (10)$$

In (10), $\overline{\alpha}_1^{(m)}(\kappa_n)$ stands for $\overline{\alpha}_1(k_1, k_2^{(m)}, \kappa_n)$. We write

$$\begin{aligned} \vec{u}^{(m)V} &= \vec{u}^{(0)(m)V} + \vec{u}^{A_n(m)V} \\ (\sigma)^{(m)V} &= (\sigma)^{(0)(m)V} + (\sigma)^{A_n(m)V} \end{aligned} \quad (11)$$

introducing subsequently the notation $\vec{u}^{(0)(m)A}$, $\vec{u}^{A_n(m)A}$, $\vec{u}^{(0)(m)B}$, $\vec{u}^{A_n(m)B}$ and even for the stress. $\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$ are x_3 – independent; $\vec{u}^{A_n(m)V}$ and $(\sigma)^{A_n(m)V}$ are proportional to the sinusoid $A_n(x_3)$ or to its spatial derivative $\partial A_n / \partial x_3$. Here also, for points P_S on the interface, $\Delta\vec{u}^V$ and $(\Delta\sigma)^V$ are expanded up to terms of first order with respect to $x_2 = \xi$ in a similar manner as in (A.2) (see Appendix A) for $\Delta\vec{u}^\infty$ and $(\Delta\sigma)^\infty$. Requiring $\Delta\vec{u}^V = \Delta\vec{u}^\infty$ and $(\Delta\sigma)^V = (\Delta\sigma)^\infty$ lead to the following equations, writing first the conditions corresponding to $k_3 = 0$ (i.e. $\Delta u_i^{(0)V} = \Delta u_i^{(0)\infty}$ and $\Delta\sigma_{ij}^{(0)V} = \Delta\sigma_{ij}^{(0)\infty}$).

$$\Delta u_1^{(0)V} = \Delta u_1^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} - \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + 4\left(\frac{(1-\nu_2)\overline{\beta}_1^{(2)}}{\mu_2} + \frac{(1-\nu_1)\overline{\beta}_1^{(1)}}{\mu_1}\right) = 0 \qquad (a)$$

$$|k_1|\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} + \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + \frac{(5-4\nu_2)\overline{\beta}_1^{(2)}}{\mu_2} - \frac{(5-4\nu_1)\overline{\beta}_1^{(1)}}{\mu_1} = \frac{ibC_\nu}{4\pi}\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (b)$$

$$\Delta u_2^{(0)V} = \Delta u_2^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} + \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + \left(\frac{\overline{\beta}_1^{(2)}}{\mu_2} - \frac{\overline{\beta}_1^{(1)}}{\mu_1}\right) = \frac{ibC_\nu}{4\pi}\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (c)$$

$$|k_1|\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} - \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + 2\left(\frac{\overline{\beta}_1^{(2)}}{\mu_2} + \frac{\overline{\beta}_1^{(1)}}{\mu_1}\right) = 0 \qquad (d)$$

$$\Delta\sigma_{11}^{(0)V} = \Delta\sigma_{11}^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2(2-\nu_2)\overline{\beta}_1^{(2)} + 2(2-\nu_1)\overline{\beta}_1^{(1)} = 0 \qquad (e)$$

$$|k_1|\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + (5-2\nu_2)\overline{\beta}_1^{(2)} - (5-2\nu_1)\overline{\beta}_1^{(1)} = -6Q_b\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (f)$$

$$\Delta\sigma_{22}^{(0)V} = \Delta\sigma_{22}^{(0)\infty} \Rightarrow$$

$$|k_1|\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2(1-\nu_2)\overline{\beta}_1^{(2)} + 2(1-\nu_1)\overline{\beta}_1^{(1)} = 0 \qquad (g)$$

$$|k_1|\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + (3-2\nu_2)\overline{\beta}_1^{(2)} - (3-2\nu_1)\overline{\beta}_1^{(1)} = -2Q_b\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (h)$$

$$\Delta\sigma_{33}^{(0)V} = \Delta\sigma_{33}^{(0)\infty} \Rightarrow$$

$$\nu_2\overline{\beta}_1^{(2)} + \nu_1\overline{\beta}_1^{(1)} = 0 \qquad (i)$$

$$\nu_2\overline{\beta}_1^{(2)} - \nu_1\overline{\beta}_1^{(1)} = -2Q_c\frac{\operatorname{sgn}(k_1)}{k_1^2} \qquad (j)$$

$$\Delta\sigma_{12}^{(0)V} = \Delta\sigma_{12}^{(0)\infty} \Rightarrow (h) \text{ and } (e) \text{ above} \qquad (12)$$

where $C_m = b\mu_m / 2\pi(1-\nu_m)$, $Q_b = i(C_2 - C_1)/4$, $Q_c = i(\nu_2 C_2 - \nu_1 C_1)/4$; $\operatorname{sgn}(k_1) = k_1/|k_1|$; μ_m and ν_m are shear modulus and Poisson's ratio. In equations (12 *a* to *j*) above, $\overline{\alpha}_1^{(m)}$ and $\overline{\beta}_1^{(m)}$ stand for $\overline{\alpha}_1(k_1, k_2^{(m)}, k_3 = 0)$ and $\overline{\beta}_1(k_1, k_2^{(m)}, k_3 = 0)$, respectively. Other elastic field components are zero. The conditions corresponding to $\Delta u_i^{A_nV} = \Delta u_i^{A_n\infty}$ and $\Delta\sigma_{ij}^{A_nV} = \Delta\sigma_{ij}^{A_n\infty}$ are now listed as :

$$\Delta u_1^{A_nV} = \Delta u_1^{A_n\infty} \Rightarrow$$

$$k_1^2\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} - \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + 4\sqrt{k_1^2 + \kappa_n^2}\left(\frac{(1-\nu_2)\overline{\beta}_1^{(2)}}{\mu_2} + \frac{(1-\nu_1)\overline{\beta}_1^{(1)}}{\mu_1}\right)$$

$$= -\frac{bC_\nu\xi_n}{8\pi}\frac{k_1\left(k_1^2 + 2\kappa_n^2\right)}{\left(k_1^2+\kappa_n^2\right)^{3/2}} \qquad (a)$$

$$k_1^2\sqrt{k_1^2+\kappa_n^2}\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} + \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + \frac{\overline{\beta}_1^{(2)}}{\mu_2}\left[(5-4\nu_2)k_1^2 + 4(1-\nu_2)\kappa_n^2\right]$$

$$-\frac{\overline{\beta}_1^{(1)}}{\mu_1}\left[(5-4\nu_1)k_1^2 + 4(1-\nu_1)\kappa_n^2\right] = 0 \qquad (b)$$

$$\Delta u_2^{A_nV} = \Delta u_2^{A_n\infty} \Rightarrow$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} + \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + \frac{\overline{\beta}_1^{(2)}}{\mu_2} - \frac{\overline{\beta}_1^{(1)}}{\mu_1} = 0 \qquad (c)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\frac{\overline{\alpha}_1^{(2)}}{\mu_2} - \frac{\overline{\alpha}_1^{(1)}}{\mu_1}\right) + 2\left(\frac{\overline{\beta}_1^{(2)}}{\mu_2} + \frac{\overline{\beta}_1^{(1)}}{\mu_1}\right) = -\frac{bC_\nu\xi_n}{8\pi}\frac{\left(3k_1^2 + 2\kappa_n^2\right)}{k_1\left(k_1^2+\kappa_n^2\right)} \qquad (d)$$

$$\Delta u_3^{A_nV} = \Delta u_3^{A_n\infty} \Rightarrow (c) \text{ above and}$$

$$\frac{\overline{\alpha}_1^{(2)}}{\mu_2} - \frac{\overline{\alpha}_1^{(1)}}{\mu_1} = -\frac{bC_\nu\xi_n}{8\pi}\frac{\left(k_1^2 + 2\kappa_n^2\right)}{k_1\left(k_1^2+\kappa_n^2\right)^{3/2}} \qquad (e)$$

$$\Delta\sigma_{11}^{A_nV} = \Delta\sigma_{11}^{A_n\infty} \Rightarrow$$
$$k_1^2\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2\sqrt{k_1^2+\kappa_n^2}\left[(2-\nu_2)\overline{\beta}_1^{(2)} + (2-\nu_1)\overline{\beta}_1^{(1)}\right]$$
$$= -\frac{iQ_b\xi_n k_1(3k_1^2+4\kappa_n^2)}{\left(k_1^2+\kappa_n^2\right)^{3/2}} \qquad (f)$$

$$k_1^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + \left[(5-2\nu_2)k_1^2 + 2(2-\nu_2)\kappa_n^2\right]\overline{\beta}_1^{(2)}$$
$$-\left[(5-2\nu_1)k_1^2 + 2(2-\nu_1)\kappa_n^2\right]\overline{\beta}_1^{(1)} = 0 \qquad (g)$$

$$\Delta\sigma_{22}^{A_nV} = \Delta\sigma_{22}^{A_n\infty} \Rightarrow$$
$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2\left[(1-\nu_2)\overline{\beta}_1^{(2)} + (1-\nu_1)\overline{\beta}_1^{(1)}\right]$$
$$= -\frac{i\xi_n(Q_b k_1^2 + 2Q_c\kappa_n^2)}{k_1\left(k_1^2+\kappa_n^2\right)} \qquad (h)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + (3-2\nu_2)\overline{\beta}_1^{(2)} - (3-2\nu_1)\overline{\beta}_1^{(1)} = 0 \qquad (i)$$

$$\Delta\sigma_{33}^{A_nV} = \Delta\sigma_{33}^{A_n\infty} \Rightarrow$$
$$\kappa_n^2\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2\sqrt{k_1^2+\kappa_n^2}\left(\nu_2\overline{\beta}_1^{(2)} + \nu_1\overline{\beta}_1^{(1)}\right)$$
$$= -\frac{i\xi_n\left\{k_1^2\left[2Q_c k_1^2 + (4Q_c - Q_b)\kappa_n^2\right] + 2Q_c\kappa_n^4\right\}}{k_1\left(k_1^2+\kappa_n^2\right)^{3/2}} \qquad (j)$$

$$\kappa_n^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + \left[2\nu_2 k_1^2 + (1+2\nu_2)\kappa_n^2\right]\overline{\beta}_1^{(2)}$$
$$-\left[2\nu_1 k_1^2 + (1+2\nu_1)\kappa_n^2\right]\overline{\beta}_1^{(1)} = 0 \qquad (k)$$

$$\Delta\sigma_{12}^{A_nV} = \Delta\sigma_{12}^{A_n\infty} \Rightarrow$$
$$k_1^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + \left[(3-2\nu_2)k_1^2 + 2(1-\nu_2)\kappa_n^2\right]\overline{\beta}_1^{(2)}$$
$$-\left[(3-2\nu_1)k_1^2 + 2(1-\nu_1)\kappa_n^2\right]\overline{\beta}_1^{(1)} = 0 \qquad (l)$$

$$k_1^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2\left[(2-\nu_2)k_1^2 + (1-\nu_2)\kappa_n^2\right]\overline{\beta}_1^{(2)}$$
$$+ 2\left[(2-\nu_1)k_1^2 + (1-\nu_1)\kappa_n^2\right]\overline{\beta}_1^{(1)} = -\frac{iQ_b\xi_n k_1(3k_1^2+2\kappa_n^2)}{k_1^2+\kappa_n^2} \qquad (m)$$

$\Delta\sigma_{13}^{A_nV} = \Delta\sigma_{13}^{A_n\infty} \Rightarrow$ (*l*) above and

$$k_1^2\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2\sqrt{k_1^2 + \kappa_n^2}\left[(1-\nu_2)\overline{\beta}_1^{(2)} + (1-\nu_1)\overline{\beta}_1^{(1)}\right]$$
$$= -\frac{iQ_b \xi_n k_1 (k_1^2 + 2\kappa_n^2)}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} \qquad (n)$$

$$\Delta\sigma_{23}^{A_n V} = \Delta\sigma_{23}^{A_n \infty} \Rightarrow$$
$$\sqrt{k_1^2 + \kappa_n^2}\left(\overline{\alpha}_1^{(2)} + \overline{\alpha}_1^{(1)}\right) + \overline{\beta}_1^{(2)} - \overline{\beta}_1^{(1)} = 0 \qquad (o)$$

$$\sqrt{k_1^2 + \kappa_n^2}\left(\overline{\alpha}_1^{(2)} - \overline{\alpha}_1^{(1)}\right) + 2\left[\overline{\beta}_1^{(2)} + \overline{\beta}_1^{(1)}\right]$$
$$= -\frac{i\xi_n\left[(2Q_c + Q_b)k_1^2 + 2Q_c\kappa_n^2\right]}{k_1\left(k_1^2 + \kappa_n^2\right)} \qquad (p) \quad (13)$$

Next, we are concerned with satisfying boundary conditions : (12) leads to the displacement and stress fields due to an interface straight edge dislocation ($\vec{b}_{II} = (b,0,0)$) parallel to the x_3 – direction at the origin; the interface is the Ox_1x_3 – plane. (13) provides the complementary terms (to first order in ξ_n) in the elastic fields of an interfacial sinusoidal climb-type edge dislocation.

III - CALCULATION RESULTS

III-1. Displacement and stress fields of an interface straight edge dislocation

Four distinct couples of values for $(\overline{\alpha}_1^{(m)}, \overline{\beta}_1^{(m)})$ are extracted from (12). These are (j= a to d) :

$$\overline{\alpha}_1^{(m)} = \frac{\overline{\overline{\alpha}}_j^{(m)}}{k_1^3} \equiv \overline{\alpha}_{1j}^{(m)}$$
$$\overline{\beta}_1^{(m)} = \overline{\overline{\beta}}_j^{(m)} \frac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \overline{\beta}_{1j}^{(m)} \qquad (14)$$

where

$$\overline{\overline{\alpha}}_a^{(m)} = \frac{iD_m C_\nu}{4}, \; \overline{\overline{\beta}}_a^{(m)} = 0 \; ;$$
$$\overline{\overline{\alpha}}_b^{(m)} = \frac{(5-6\nu_m)Q_b}{4}, \; \overline{\overline{\beta}}_b^{(m)} = (-1)^{m-1}\frac{3Q_b}{4} \; ;$$

$$\overline{\overline{\alpha}}_c^{(m)} = \frac{1}{2}\left(\frac{iC_\nu D_m}{2} + \frac{\mu_2 Q_c}{\mu_1 \nu_1}\delta_{m2} + \frac{\mu_1 Q_c}{\mu_2 \nu_2}\delta_{m1} + \frac{(9-8\nu_m)Q_c}{\nu_m}\right),\ \overline{\overline{\beta}}_c^{(m)} = (-1)^{m-1}\frac{Q_c}{\nu_m};$$

$$\overline{\overline{\alpha}}_d^{(m)} = \frac{1}{2}\left(\frac{iC_\nu D_m}{2} - \frac{\mu_2 Q_c}{\mu_1 \nu_1}\delta_{m2} - \frac{\mu_1 Q_c}{\mu_2 \nu_2}\delta_{m1} + \frac{3Q_c}{\nu_m}\right),\ \overline{\overline{\beta}}_d^{(m)} = \overline{\overline{\beta}}_c^{(m)} \qquad (15)$$

where δ_{ij} is the Kronecker delta and $D_m = b\mu_m / 2\pi = C_m(1-\nu_m)$. The couples $(\overline{\alpha}_{1j}^{(m)}, \overline{\beta}_{1j}^{(m)})$, *j*=*a* to *d*, are obtained from (12*a* to *d*) associated with the displacement, (12 *e* to *h*) associated with stresses, (12 *a*, *b*, *i*, *j*) and (12 *c*, *d*, *i*, *j*), respectively. None of these values satisfies the entire (12). The associated elastic fields denoted $\vec{u}_{a\ \text{to}\ d}^{(0)(m)V}$ and $(\sigma)_{a\ \text{to}\ d}^{(0)(m)V}$ are displayed below. A superposition of these partial fields will provide the complete form of the solution. We have at position $\vec{x} = (x_1, x_2, x_3)$ ($\vec{u}^{(0)(m)V} \equiv \vec{u}_j^{(0)(m)V}$, $(\sigma)^{(0)(m)V} \equiv (\sigma)_j^{(0)(m)V}$; *j*=*a* to *d*):

$$u_{1j}^{(0)(m)V} = \frac{i\left(\overline{\overline{\alpha}}_j^{(m)} + (-1)^m 4(1-\nu_m)\overline{\overline{\beta}}_j^{(m)}\right)}{\mu_m}\tan^{-1}\frac{x_1}{|x_2|} + \frac{i\overline{\overline{\beta}}_j^{(m)}}{\mu_m}\frac{x_1 x_2}{r^2},$$

$$u_{2j}^{(0)(m)V} = \frac{i\left((-1)^m\overline{\overline{\alpha}}_j^{(m)} + \overline{\overline{\beta}}_j^{(m)}\right)}{\mu_m}\ln|x_1|\delta_A(x_2) - \frac{i(-1)^m\overline{\overline{\beta}}_j^{(m)}}{\mu_m}\frac{x_2^2}{r^2},$$

$$\sigma_{11j}^{(0)(m)V} = 2i\left(\overline{\overline{\alpha}}_j^{(m)} + (-1)^m 2(2-\nu_m)\overline{\overline{\beta}}_j^{(m)}\right)\left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right)$$

$$-\frac{2i\overline{\overline{\beta}}_j^{(m)}x_2(x_1^2 - x_2^2)}{r^4},$$

$$\sigma_{22j}^{(0)(m)V} = -2i\left(\overline{\overline{\alpha}}_j^{(m)} + (-1)^m 2(1-\nu_m)\overline{\overline{\beta}}_j^{(m)}\right)\left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right)$$

$$+\frac{2i\overline{\overline{\beta}}_j^{(m)}x_2(x_1^2 - x_2^2)}{r^4},$$

$$\sigma_{33j}^{(0)(m)V} = (-1)^m 4\nu_m i\overline{\overline{\beta}}_j^{(m)}\left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right),$$

$$\sigma_{12j}^{(0)(m)V} = 2i\left((-1)^m\overline{\overline{\alpha}}_j^{(m)} + (3-2\nu_m)\overline{\overline{\beta}}_j^{(m)}\right)\left(\frac{x_1}{r^2} + (\overline{J}_1 - \frac{x_1}{r^2})\delta_A(x_2)\right)$$

$$+\frac{(-1)^m 4i\overline{\overline{\beta}}_j^{(m)}\operatorname{sgn}(x_2)x_1x_2^2}{r^4}; \qquad (16)$$

$$\overline{J}_1 = \int_0^\infty \sin k_1 x_1 dk_1 .$$

Here, $\operatorname{sgn}(x_i) = x_i / |x_i|$, $\delta(x_1)$ is the Dirac delta function and δ_A has the following definition: $\delta_A(x_2) = 0$ when $x_2 \neq 0$ and $\delta_A(x_2) = 1$ when $x_2 = 0$; $r^2 = x_1^2 + x_2^2$. Constant terms are omitted in the displacement. The other elastic fields are zero. We define the elastic fields $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ of an interface straight edge dislocation as

$$\begin{aligned} \vec{u}^{(0)(m)} &= \vec{u}^{(0)(m)\infty} - \vec{u}^{(0)(m)W} \\ (\sigma)^{(0)(m)} &= (\sigma)^{(0)(m)\infty} - (\sigma)^{(0)(m)W} \end{aligned} \quad (17)$$

with

$$\begin{aligned} \vec{u}^{(0)(m)W} &= \sum_{j=a \text{ to } d} \eta_j^{(m)} \vec{u}_j^{(0)(m)V} \\ (\sigma)^{(0)(m)W} &= \sum_{j=a \text{ to } d} \eta_j^{(m)} (\sigma)_j^{(0)(m)V} . \end{aligned} \quad (18)$$

Again $\vec{u}^{(0)(m)\infty}$ and $(\sigma)^{(0)(m)\infty}$ are due to a straight edge $\vec{b}_{II} = (b,0,0)$ parallel to the x_3 – direction at the origin in an infinite medium (see [7] for example); $\vec{u}_{a \text{ to } d}^{(0)(m)V}$ and $(\sigma)_{a \text{ to } d}^{(0)(m)V}$ are given in (16). $\eta_{a \text{ to } d}^{(m)}$ are real, to be determined by the condition that the elastic fields satisfy the following relations :

- $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ are continuous when crossing the Ox_1x_3 – plane.
- $\oint_\Gamma du_1^{(0)(m)} = b$ for a closed contour Γ in x_1x_2 encircling the dislocation.
- $\vec{u}^{(0)(m)W}$ vanishes far from the interface (i.e. when $|x_2| \to \infty$).

It is easy to show that the second condition above correspond to $u_1^{(0)(m)}$ constant with *m* on the interface. All the stresses involved in $(\sigma)^{(0)(m)\infty}$ and $(\sigma)_{a \text{ to } d}^{(0)(m)V}$ vanish at infinity. Under such conditions, $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ correspond to an interface straight edge dislocation. Next, we express the quantities involved in the above-mentioned requirements and proceed to satisfy these.

$$u_1^{(0)(1)}(x_1, x_2 = 0, x_3) = u_1^{(0)(2)}(x_1, x_2 = 0, x_3) \Rightarrow$$

$$\frac{1}{\mu_m}\left(\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)} + 4(1-\nu_m)e_7(m)\right) \equiv e_1;$$

$$u_2^{(0)(1)} = u_2^{(0)(2)} \Rightarrow$$

$$\frac{1}{\mu_m}\left(C_m(2\nu_m - 1) - 2i(-1)^m\left[\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)} + e_7(m)\right]\right) \equiv e_2;$$

$$\sigma_{11}^{(0)(1)} = \sigma_{11}^{(0)(2)} \Rightarrow$$

$$\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)} + 2(2-\nu_m)e_7(m) \equiv e_3 \,;$$

$$\sigma_{22}^{(0)(1)} = \sigma_{22}^{(0)(2)} \Rightarrow$$
$$\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)} + 2(1-\nu_m)e_7(m) \equiv e_4 \,;$$

$$\sigma_{33}^{(0)(1)} = \sigma_{33}^{(0)(2)} \Rightarrow$$
$$\nu_m e_7(m) \equiv e_5 \,;$$

$$\sigma_{12}^{(0)(1)} = \sigma_{12}^{(0)(2)} \Rightarrow$$
$$C_m - 2i(-1)^m \left(\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)} + (3-2\nu_m)e_7(m) \right) \equiv e_6 \qquad (19)$$

where

$$(-1)^m \sum_j \eta_j^{(m)} \overline{\overline{\beta}}_j^{(0)(m)} \equiv e_7(m) \,.$$

Here, the subscript *j* takes the values *a* to *d*. The continuity of an elastic field requires associated e_i constant with *m* at the crossing of the interface. The additional condition, $\vec{u}^{(0)(m)}$ tends to $\vec{u}^{(0)(m)\infty}$ when $|x_2| \to \infty \Rightarrow$, corresponds to vanishing $\vec{u}^{(0)(m)W}(\vec{x})$ far from the interface. This gives

$$\vec{u}^{(0)(m)W}(\vec{x}) \text{ vanishes when } |x_2| \to \infty \Rightarrow$$
$$u_2^{(0)(m)W} = -ie_7(m)/\mu_m = 0 \,. \qquad (20)$$

This condition is due to the x_2 – component of the displacement $\vec{u}^{(0)(m)W}(\vec{x})$ only; the other x_1 – component tends to zero. Because $e_7(m)$ is a constant in medium (*m*), imposing its value to be identically zero is not mandatory; elastic displacements, differing only by a constant, are admissible in elasticity. Instead, we shall choose a value to $e_7(m)$ that ensures strongest continuity of the elastic fields at the crossing of the interface. We have checked that $e_7(m) \equiv 0$ leads to poor continuity (i.e. only a restricted number of elastic field components, $u_i^{(0)(m)}$ and $\sigma_{ij}^{(0)(m)}$, are continuous when crossing the interface). A best continuity behaviour of the elastic fields at the interface is presented as follows. The relative volume variation $(\Delta V/V)^{(0)(m)}$ at arbitrary position $(x_1, 0, x_3)$ is obtained to be

$$\left(\frac{\Delta V}{V}\right)^{(0)(m)}(x_1, 0, x_3) = -2\pi\delta(x_1) i V_R^{(0)} \tag{21}$$

with

$$V_R^{(0)} = \frac{1-2\nu_m}{\mu_m} e_7(m) = (1-2\nu_m) i u_2^{(0)(m)W}(x_1, \infty, x_3)\ .$$

It has a singularity given by the Dirac delta function $\delta(x_1)$. We demand $V_R^{(0)}$ to be constant with m = 1 and 2; this gives a value to $e_7(m)$ that is m dependent. We have

$$\frac{e_7(2)}{e_7(1)} = \Gamma\,\frac{1-2\nu_1}{1-2\nu_2} \tag{22}$$

where $\Gamma = \mu_2/\mu_1$. The expressions for $e_1(m)$ and $e_6(m)$ in (19) are merged to read

$$C_m - 2i(-1)^m \mu_m \left[e_1(m) - V_R^{(0)}\right] = e_6(m)\ . \tag{23}$$

We seek continuity of the elastic fields on the interface, hence we impose $e_1(m)$ and $e_6(m)$ to be constant with m; this leads, with the help of (23), to

$$e_6 = \frac{\mu_1 C_2 - \mu_2 C_1}{\mu_1 - \mu_2},$$

$$e_1 = \frac{i(C_2 - C_1)}{2(\mu_1 - \mu_2)} + V_R^{(0)}\ . \tag{24}$$

These expressions are unchanged by inverting the elastic constants. We isolate $\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)}$ with the expression for e_1 in (19) and obtain

$$\sum_j \eta_j^{(m)} \overline{\overline{\alpha}}_j^{(0)(m)} = \frac{i\mu_m (C_2 - C_1)}{2(\mu_2 - \mu_1)} - \frac{(3-2\nu_m)\mu_m}{1-2\nu_m} V_R^{(0)}\ . \tag{25}$$

Introducing (25) in the expression for $e_2(m)$ in (19) and imposing $e_2(m)$ to be constant with m, provide a value to $V_R^{(0)}$ in the form

$$V_R^{(0)} = -\frac{ib\,(1-2\nu_1)(1-2\nu_2)}{8\pi(1-\nu_1)(1-\nu_2)}\ . \tag{26}$$

This relation is unchanged by inverting the elastic constants. The other elastic parameters are obtained as

$$e_2 = -\frac{b}{\pi} + \frac{4i\left[\mu_1(1-\nu_1)(1-2\nu_2) - \mu_2(1-\nu_2)(1-2\nu_1)\right]}{(1-2\nu_1)(1-2\nu_2)(\mu_2-\mu_1)} V_R^{(0)},$$
$$e_3(m) = \frac{i\mu_m(C_2 - C_1)}{2(\mu_2-\mu_1)} + \frac{\mu_m}{1-2\nu_m} V_R^{(0)},$$
$$e_4(m) = \frac{i\mu_m(C_2 - C_1)}{2(\mu_2-\mu_1)} - \frac{\mu_m}{1-2\nu_m} V_R^{(0)},$$
$$e_5(m) = \frac{\nu_m \mu_m}{1-2\nu_m} V_R^{(0)}. \tag{27}$$

The continuities at the interface of $u_1^{(0)(m)}$, $u_2^{(0)(m)}$, $\sigma_{12}^{(0)(m)}$ and $(\Delta V/V)^{(0)(m)}$ are achieved. The elastic fields, at position $\vec{x} = (x_1, x_2, x_3)$, are :

$$u_1^{(0)(m)} = \frac{b}{2\pi}\tan^{-1}\frac{x_2}{x_1} - ie_1 \tan^{-1}\frac{x_1}{|x_2|} + \frac{x_1 x_2}{r^2}\left(\frac{C_m}{2\mu_m} + (-1)^{m-1}\frac{1}{1-2\nu_m} iV_R^{(0)}\right),$$

$$u_2^{(0)(1)} = -\frac{(1-2\nu_1)C_1}{2\mu_1}\ln r - 2\ln|x_1|\delta_A(x_2)\left(\frac{C_2 - C_1}{4(\mu_2-\mu_1)} + \frac{1-\nu_1}{1-2\nu_1} iV_R^{(0)}\right)$$
$$+\frac{x_2^2}{r^2}\left(\frac{C_1}{2\mu_1} + \frac{1}{1-2\nu_1} iV_R^{(0)}\right),$$

$$\sigma_{11}^{(0)(m)} = -C_m \frac{x_2(x_2^2 + 3x_1^2)}{r^4} - \left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right) 2ie_3(m)$$
$$+\frac{x_2(x_1^2 - x_2^2)}{r^4}(-1)^m \frac{\mu_m}{1-2\nu_m} 2iV_R^{(0)},$$

$$\sigma_{22}^{(0)(m)} = \frac{x_2(x_1^2 - x_2^2)}{r^4}\left[C_m + (-1)^m \frac{\mu_m 2iV_R^{(0)}}{1-2\nu_m}\right] + \left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right) 2ie_4(m),$$

$$\sigma_{33}^{(0)(m)} = -2\nu_m C_m \frac{x_2}{r^2} - \left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right)\frac{\nu_m\mu_m}{1-2\nu_m} 4iV_R^{(0)},$$

$$\sigma_{12}^{(0)(1)} = C_1 \frac{x_1(x_1^2 - x_2^2)}{r^4} - \left(\frac{x_1}{r^2} + (\bar{J}_1 - \frac{x_1}{r^2})\delta_A(x_2)\right)\frac{\mu_1(C_2 - C_1)}{\mu_2 - \mu_1}$$
$$-\operatorname{sgn}(x_2)\frac{x_1 x_2^2}{r^4}\frac{\mu_1}{1-2\nu_1} 4iV_R^{(0)},$$

$$\left(\frac{\Delta V}{V}\right)^{(0)(m)} = \frac{1-2\nu_m}{2(1+\nu_m)\mu_m}\left(\sigma_{11}^{(0)(m)} + \sigma_{22}^{(0)(m)} + \sigma_{33}^{(0)(m)}\right)$$

$$= -\left(\frac{x_2}{r^2} + \pi\delta(x_1)\delta_A(x_2)\right)2iV_R^{(0)} + \frac{x_2(x_1^2 - x_2^2)}{r^4}(-1)^m \frac{2iV_R^{(0)}}{1+\nu_m}$$

$$-\frac{(1-2\nu_m)C_m}{\mu_m}\frac{x_2}{r^2}. \tag{28}$$

Corresponding expressions for half-space $m = 2$ are obtained by inverting the elastic constants. The relations between $e_i(m)$ and $V_R^{(0)}$ are taken from relations (21) and (27); the value of $V_R^{(0)}$ is given by (26). Other elastic fields are zero. Dirac delta function $\delta(x_1)$ is present in $\sigma_{ii}^{(0)(m)}(x_1, 0, x_3)$ (i=1 to 3) and $(\Delta V/V)^{(0)(m)}$, on the interface. The only stress component with a singularity $1/x_1$ on the interface is $\sigma_{12}^{(0)(m)}$.

III-2. Elastic fields of an interface climb-type sinusoidal edge dislocation

Five values for ($\overline{\alpha}_1^{(m)}(\kappa_n)$, $\overline{\beta}_1^{(m)}(\kappa_n)$) are extracted from (13); these are

a) $$\overline{\alpha}_1^{(m)} = \frac{\xi_n l_{1a}^{(m)}}{k_1\sqrt{k_1^2+\kappa_n^2}}\left(1+\frac{\kappa_n^2}{k_1^2+\kappa_n^2}\right) \equiv \overline{\alpha}_{1a}^{A_n(m)}, \quad \overline{\beta}_1^{(m)} = 0 \equiv \overline{\beta}_{1a}^{A_n(m)};$$

b) $$\overline{\alpha}_1^{(m)} = \frac{\xi_n l_{1b}^{(m)}}{k_1\sqrt{k_1^2+\kappa_n^2}}\left(l_{2b}^{(m)} + \frac{\kappa_n^2 l_{3b}^{(m)}}{k_1^2+\kappa_n^2} - \frac{\kappa_n^2 l_{4b}^{(m)}}{k_1^2+\tilde{\Omega}_{1b}\kappa_n^2}\right) \equiv \overline{\alpha}_{1b}^{A_n(m)},$$

$$\overline{\beta}_1^{(m)} = \xi_n t_{1b}^{(m)} k_1\left(\frac{1}{k_1^2+\kappa_n^2} - \frac{\tilde{\Omega}_{1b}}{k_1^2+\tilde{\Omega}_{1b}\kappa_n^2}\right) \equiv \overline{\beta}_{1b}^{A_n(m)};$$

c) $$\overline{\alpha}_1^{(m)} = \frac{\xi_n l_{1c}^{(m)}}{k_1\sqrt{k_1^2+\kappa_n^2}}\left(1-2\nu_m - \frac{\kappa_n^2}{k_1^2+\kappa_n^2}\right) \equiv \overline{\alpha}_{1c}^{A_n(m)}$$

$$\overline{\beta}_1^{(m)} = -\frac{\xi_n iQ_b}{2}\frac{k_1}{k_1^2+\kappa_n^2} \equiv \overline{\beta}_{1c}^{A_n}$$ independent of m;

d) $$\overline{\alpha}_1^{(m)} = \frac{\xi_n l_{1d}^{(m)}}{k_1\sqrt{k_1^2+\kappa_n^2}}\left(l_{2d}^{(m)} - \frac{\kappa_n^2 l_{3d}^{(m)}}{k_1^2+\kappa_n^2}\right) \equiv \overline{\alpha}_{1d}^{A_n(m)},$$

$$\overline{\beta}_1^{(m)} = \xi_n t_{1d}^{(m)}\frac{k_1}{k_1^2+\kappa_n^2} \equiv \overline{\beta}_{1d}^{A_n(m)};$$

e) $$\overline{\alpha}_1^{(m)} = \frac{\xi_n l_{1e}^{(m)} k_1}{\sqrt{k_1^2+\kappa_n^2}}\left(\frac{l_{2e}^{(m)}}{k_1^2 - r_2^*\kappa_n^2} + \frac{l_{3e}^{(m)}}{k_1^2 - r_1^*\kappa_n^2} - \frac{l_{4e}^{(m)}}{k_1^2 - r_m^*\kappa_n^2}\right.$$

$$- \frac{2Q_b}{k_1^2 + \kappa_n^2} + \frac{4Q_c}{k_1^2}\Bigg) \equiv \overline{\alpha}_{1e}^{A_n(m)}$$

$$\overline{\beta}_1^{(m)} = -\frac{i\xi_n k_1}{2}\left(\frac{t_{1e}^{(m)}}{k_1^2 - r_m^* \kappa_n^2} + \frac{Q_b}{k_1^2 + \kappa_n^2}\right) \equiv \overline{\beta}_{1e}^{A_n(m)} \tag{29}$$

where

$l_{1a}^{(m)} = (-1)^{m-1}\mu_m bC_\nu / 16\pi$, $C_\nu = [1/(1-\nu_1) \ - 1/(1-\nu_2)]$, $r_m^* = (1-\nu_m)/\nu_m$,

$\rho_m = \nu_1\delta_{m2} + \nu_2\delta_{m1}$, $\tilde{\Omega}_{1b} = 4(1-\nu_1)(1-\nu_2)/[(1-\nu_1)(1-2\nu_2) + (1-\nu_2)(1-2\nu_1)]$;

$l_{1b}^{(m)} = (-1)^{m-1} l_{1a}^{(m)} / 4(1-\nu_1)(1-\nu_2)(1-\tilde{\Omega}_{1b})$,

$l_{2b}^{(m)} = (1-\tilde{\Omega}_{1b})[(\nu_1-\nu_2)\tilde{\Omega}_{1b} + (-1)^{m-1}4(1-\nu_1)(1-\nu_2)(1+2\tilde{\Omega}_{1b})]$,

$l_{3b}^{(m)} = (-1)^{m-1}4(1-\nu_1)(1-\nu_2)(1-\tilde{\Omega}_{1b}) - (\nu_1-\nu_2)\tilde{\Omega}_{1b}$,

$l_{4b}^{(m)} = \tilde{\Omega}_{1b}^2[(-1)^{m-1}8(1-\nu_1)(1-\nu_2)(1-\tilde{\Omega}_{1b}) - (\nu_1-\nu_2)\tilde{\Omega}_{1b}]$,

$t_{1b}^{(m)} = 2\tilde{\Omega}_{1b}(1-\rho_m) l_{1b}^{(m)}$;

$l_{1c}^{(m)} = (-1)^m iQ_b / 2$;

$l_{1d}^{(m)} = (-1)^m i / \ 2[\nu_2(1-\nu_1) + \nu_1(1-\nu_2)]$,

$l_{2d}^{(m)} = Q_c(4 - 3\rho_m - \nu_m - 2[\nu_2(1-\nu_1) + \nu_1(1-\nu_2)]) - Q_b[\nu_2(1-\nu_1) + \nu_1(1-\nu_2)]$,

$l_{3d}^{(m)} = Q_c(4 - 3\rho_m - \nu_m - 2) - Q_b[\nu_2(1-\nu_1) + \nu_1(1-\nu_2)]$,

$t_{1d}^{(m)} = (-1)^{m-1}2Q_c(1-\rho_m) l_{1d}^{(m)}$;

$l_{1e}^{(m)} = (-1)^{m-1} i / 4$, $l_{2e}^{(m)} = (\delta_{m1}/\nu_2)[Q_c - Q_b/(1+r_2^*)]$,

$l_{3e}^{(m)} = (\delta_{m2}/\nu_1)[Q_c - Q_b/(1+r_1^*)]$, $l_{4e}^{(m)} = (5/\nu_m)[Q_c - Q_b/(1+r_m^*)]$,

$t_{1e}^{(m)} = (1/\nu_m)[Q_c - Q_b/(1+r_m^*)]$. (30)

None of these couples satisfies the entire conditions (13). For each couple, we give in Appendix B the associated oscillating elastic fields $\vec{u}^{A_n(m)V} = \vec{u}^{A_n(m)A} + \vec{u}^{A_n(m)B}$ and $(\sigma)^{A_n(m)V} = (\sigma)^{A_n(m)A} + (\sigma)^{A_n(m)B}$ defined in (11). A superposition of these partial fields will provide the complete form of solution (to first order in ξ_n). The elastic fields $\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$ of an interface sinusoidal edge dislocation may be written as

$$\vec{u}^{(m)} = \vec{u}^{(0)(m)} + \vec{u}^{A_n(m)}$$
$$(\sigma)^{(m)} = (\sigma)^{(0)(m)} + (\sigma)^{A_n(m)} . \tag{31}$$

$\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$ (28) correspond to the fields of a straight edge dislocation lying on a planar interface; $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ are oscillating expressions proportional to either the sinusoid $A_n(x_3)$ or its spatial derivative $\partial A_n / \partial x_3$ in the forms

$$\vec{u}^{A_n(m)} = \vec{u}^{A_n(m)\infty} - \vec{u}^{A_n(m)W}$$
$$(\sigma)^{A_n(m)} = (\sigma)^{A_n(m)\infty} - (\sigma)^{A_n(m)W} \quad (32)$$

with

$$\vec{u}^{A_n(m)W} = \sum_{j=a \text{ to } e} \eta_j^{A_n(m)} \vec{u}_j^{A_n(m)V}$$
$$(\sigma)^{A_n(m)W} = \sum_{j=a \text{ to } e} \eta_j^{A_n(m)} (\sigma)_j^{A_n(m)V} . \quad (33)$$

$\vec{u}_{a \text{ to } e}^{A_n(m)V}$ and $(\sigma)_{a \text{ to } e}^{A_n(m)V}$ are given in Appendix B (for $\vec{u}^{A_n(m)\infty}$ and $(\sigma)^{A_n(m)\infty}$, see [7]) ; $\eta_{a \text{ to } e}^{A_n(m)}$ are real to be determined by the requirement that the elastic fields be continuous when crossing the interface. It is sufficient to write this condition for points on the average interface plane. Before displaying the corresponding equations, we stress what follows.

- Both $u_1^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{13}^{A_n(m)}(x_1,0,x_3)$ contain terms with singularity $1/x_1$ only; the associated coefficients $e_1^{A_n}$ and $e_8^{A_n}$, respectively, have been taken constant.
- $u_2^{A_n(m)}(x_1,0,x_3)$, $\sigma_{12}^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{23}^{A_n(m)}(x_1,0,x_3)$ are bounded functions. Under such conditions, we have considered their linear forms with respect to x_1 and posed the terms proportional to x_1 constant with m=1 and 2.
- $\sigma_{11}^{A_n(m)}(x_1,0,x_3)$, $\sigma_{22}^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{33}^{A_n(m)}(x_1,0,x_3)$ have terms with singularities $1/x_1^2$ and $\ln|x_1|$; the associated coefficients ($e_{41}^{A_n}$, $e_{51}^{A_n}$ and $e_{61}^{A_n}$) and ($e_{42}^{A_n}$, $e_{52}^{A_n}$ and $e_{62}^{A_n}$), respectively, have been set constant.
- $u_3^{A_n(m)}(x_1,0,x_3)$ contains a term with the singularity $\ln|x_1|$ only. The associated coefficients $e_3^{A_n}$ is taken constant.

We may write :

$$u_1^{A_n(1)}(x_1,0,x_3) = u_1^{A_n(2)}(x_1,0,x_3) \;\; \Rightarrow$$

$$\frac{1}{\mu_m}\Big\{C_m(3-2\nu_m)-\eta_a^{A_n(m)}4l_{1a}^{(m)}-\eta_b^{A_n(m)}4\Big[l_{1b}^{(m)}l_{2b}^{(m)}+(-1)^m4(1-\nu_m)t_{1b}^{(m)}(1-\tilde{\Omega}_{1b})\Big]$$
$$+\eta_c^{A_n(m)}4l_{1c}^{(m)}(3-2\nu_m)-\eta_d^{A_n(m)}4\Big[l_{1d}^{(m)}l_{2d}^{(m)}+t_{1d}^{(m)}(-1)^m4(1-\nu_m)\Big]$$
$$-\eta_e^{A_n(m)}4\Big(l_{1e}^{(m)}[l_{2e}^{(m)}+l_{3e}^{(m)}-l_{4e}^{(m)}-2Q_b+4Q_c]+(-1)^{m-1}i2(1-\nu_m)(t_{1e}^{(m)}+Q_b)\Big)\Big\}\equiv e_1^{A_n};$$

$$u_2^{A_n(1)}=u_2^{A_n(2)}\ \Rightarrow$$

$$\frac{1}{\mu_m}\Big\{\eta_a^{A_n(m)}(-1)^m2l_{1a}^{(m)}+\eta_b^{A_n(m)}2\Big[(-1)^m l_{1b}^{(m)}(l_{3b}^{(m)}-l_{4b}^{(m)})+t_{1b}^{(m)}(\tilde{\Omega}_{1b}^2-1)\Big]$$
$$+\eta_d^{A_n(m)}2\Big[(-1)^{m-1}l_{1d}^{(m)}l_{3d}^{(m)}-t_{1d}^{(m)}\Big]$$
$$+\eta_e^{A_n(m)}\Big((-1)^m2l_{1e}^{(m)}[l_{2e}^{(m)}r_2^*+l_{3e}^{(m)}r_1^*-l_{4e}^{(m)}r_m^*+2Q_b]+i(Q_b-t_{1e}^{(m)}r_m^*)\Big)\Big\}\equiv e_2^{A_n};$$

$$u_3^{A_n(1)}=u_3^{A_n(2)}\ \Rightarrow$$

$$\frac{1}{\mu_m}\Big\{C_m(1-2\nu_m)+\eta_a^{A_n(m)}4l_{1a}^{(m)}+\eta_b^{A_n(m)}4l_{1b}^{(m)}l_{2b}^{(m)}+\eta_c^{A_n(m)}4l_{1c}^{(m)}(1-2\nu_m)$$
$$+\eta_d^{A_n(m)}4l_{1d}^{(m)}l_{2d}^{(m)}+\eta_e^{A_n(m)}4l_{1e}^{(m)}[l_{2e}^{(m)}+l_{3e}^{(m)}-l_{4e}^{(m)}-2Q_b+4Q_c]\Big\}\equiv e_3^{A_n};$$

$$\sigma_{11}^{A_n(1)}=\sigma_{11}^{A_n(2)}\ \Rightarrow$$

$$3C_m-\eta_a^{A_n(m)}4l_{1a}^{(m)}-\eta_b^{A_n(m)}4\Big[l_{1b}^{(m)}l_{2b}^{(m)}+t_{1b}^{(m)}(-1)^m2(2-\nu_m)(1-\tilde{\Omega}_{1b})\Big]$$
$$+\eta_c^{A_n(m)}12\,l_{1c}^{(m)}-\eta_d^{A_n(m)}4\Big[l_{1d}^{(m)}l_{2d}^{(m)}+t_{1d}^{(m)}(-1)^m2(2-\nu_m)\Big]$$
$$-\eta_e^{A_n(m)}4\Big(l_{1e}^{(m)}[l_{2e}^{(m)}+l_{3e}^{(m)}-l_{4e}^{(m)}-2Q_b+4Q_c]+(-1)^{m-1}i(2-\nu_m)(t_{1e}^{(m)}+Q_b)\Big)\equiv e_{41}^{A_n},$$
$$-C_m+\eta_b^{A_n(m)}2\Big[l_{1b}^{(m)}(l_{2b}^{(m)}-l_{3b}^{(m)}+l_{4b}^{(m)})+(-1)^m t_{1b}^{(m)}2(2-\nu_m)(1-\tilde{\Omega}_{1b}^2)\Big]$$
$$-\eta_c^{A_n(m)}4l_{1c}^{(m)}+\eta_d^{A_n(m)}2\Big[l_{1d}^{(m)}(l_{2d}^{(m)}+l_{3d}^{(m)})+t_{1d}^{(m)}(-1)^m2(2-\nu_m)\Big]$$
$$+\eta_e^{A_n(m)}2\Big(l_{1e}^{(m)}[l_{2e}^{(m)}(1-r_2^*)+l_{3e}^{(m)}(1-r_1^*)$$
$$-l_{4e}^{(m)}(1-r_m^*)-4Q_b+4Q_c]+i(-1)^{m-1}(2-\nu_m)(Q_b-t_{1e}^{(m)}r_m^*)\Big)\equiv e_{42}^{A_n};$$

$$\sigma_{22}^{A_n(1)}=\sigma_{22}^{A_n(2)}\ \Rightarrow$$

$$-C_m+\eta_a^{A_n(m)}4l_{1a}^{(m)}+\eta_b^{A_n(m)}4\Big[l_{1b}^{(m)}l_{2b}^{(m)}+t_{1b}^{(m)}(-1)^m2(1-\nu_m)(1-\tilde{\Omega}_{1b})\Big]$$
$$-\eta_c^{A_n(m)}4l_{1c}^{(m)}+\eta_d^{A_n(m)}4\Big[l_{1d}^{(m)}l_{2d}^{(m)}+t_{1d}^{(m)}(-1)^m2(1-\nu_m)\Big]$$
$$+\eta_e^{A_n(m)}4\Big(l_{1e}^{(m)}[l_{2e}^{(m)}+l_{3e}^{(m)}-l_{4e}^{(m)}-2Q_b+4Q_c]+(-1)^{m-1}i(1-\nu_m)(t_{1e}^{(m)}+Q_b)\Big)\equiv e_{51}^{A_n},$$
$$C_m(1-2\nu_m)+\eta_a^{A_n(m)}4l_{1a}^{(m)}+\eta_b^{A_n(m)}4\Big[l_{1b}^{(m)}(l_{3b}^{(m)}-l_{4b}^{(m)})+(-1)^m t_{1b}^{(m)}2(1-\nu_m)(\tilde{\Omega}_{1b}^2-1)\Big]$$
$$+\eta_c^{A_n(m)}4l_{1c}^{(m)}(1-2\nu_m)-\eta_d^{A_n(m)}4\Big[l_{1d}^{(m)}l_{3d}^{(m)}+t_{1d}^{(m)}(-1)^m2(1-\nu_m)\Big]$$
$$+\eta_e^{A_n(m)}4\Big(l_{1e}^{(m)}[l_{2e}^{(m)}r_2^*+l_{3e}^{(m)}r_1^*-l_{4e}^{(m)}r_m^*+2Q_b]+i(-1)^m(1-\nu_m)(Q_b-t_{1e}^{(m)}r_m^*)\Big)\equiv e_{52}^{A_n};$$

$$\sigma_{33}^{A_n(1)} = \sigma_{33}^{A_n(2)} \Rightarrow$$

$$\nu_m C_m + \eta_b^{A_n(m)} 4t_{1b}^{(m)}(-1)^{m-1}\nu_m(1-\tilde{\Omega}_{1b}) + \eta_c^{A_n(m)} 4\nu_m l_{1c}^{(m)}$$
$$+\eta_d^{A_n(m)} 4t_{1d}^{(m)}(-1)^{m-1}\nu_m + \eta_e^{A_n(m)} 2i(-1)^m \nu_m (t_{1e}^{(m)} + Q_b) \equiv e_{61}^{A_n},$$
$$C_m + \eta_a^{A_n(m)} 4l_{1a}^{(m)} + \eta_b^{A_n(m)} 4\left[l_{1b}^{(m)} l_{2b}^{(m)} + (-1)^m t_{1b}^{(m)} 2\nu_m (\tilde{\Omega}_{1b}^2 - 1)\right]$$
$$+\eta_c^{A_n(m)} 4l_{1c}^{(m)} + \eta_d^{A_n(m)} 4\left[l_{1d}^{(m)} l_{2d}^{(m)} + t_{1d}^{(m)}(-1)^{m-1} 2\nu_m\right]$$
$$+\eta_e^{A_n(m)} 4\left(l_{1e}^{(m)}[l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} - 2Q_b + 4Q_c] + i(-1)^m \nu_m (Q_b - t_{1e}^{(m)} r_m^*)\right) \equiv e_{62}^{A_n};$$

$$\sigma_{12}^{A_n(1)} = \sigma_{12}^{A_n(2)} \Rightarrow$$

$$\eta_a^{A_n(m)}(-1)^m 2l_{1a}^{(m)} + \eta_b^{A_n(m)} 2\left((-1)^m l_{1b}^{(m)}(l_{3b}^{(m)} - l_{4b}^{(m)}\tilde{\Omega}_{1b}^{1/2})\right.$$
$$\left.+ t_{1b}^{(m)}[-1 - 2(1-\nu_m)\tilde{\Omega}_{1b}^{3/2} + (3 - 2\nu_m)\tilde{\Omega}_{1b}^{5/2}]\right)$$
$$+\eta_d^{A_n(m)} 2\left[(-1)^{m-1} l_{1d}^{(m)} l_{3d}^{(m)} - t_{1d}^{(m)}\right] + \eta_e^{A_n(m)} iQ_b\left[1 + (-1)^{m-1} 4il_{1e}^{(m)}\right] \equiv e_7^{A_n};$$

$$\sigma_{13}^{A_n(1)} = \sigma_{13}^{A_n(2)} \Rightarrow$$

$$-C_m + \eta_a^{A_n(m)} 4l_{1a}^{(m)} + \eta_b^{A_n(m)} 4\left[l_{1b}^{(m)} l_{2b}^{(m)} + t_{1b}^{(m)}(-1)^m 2(1-\nu_m)(1-\tilde{\Omega}_{1b})\right]$$
$$-\eta_c^{A_n(m)} 4l_{1c}^{(m)} + \eta_d^{A_n(m)} 4\left[l_{1d}^{(m)} l_{2d}^{(m)} + t_{1d}^{(m)}(-1)^m 2(1-\nu_m)\right]$$
$$+\eta_e^{A_n(m)} 4\left(l_{1e}^{(m)}[l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} + 2Q_b - 4Q_c] + (-1)^{m-1} i(1-\nu_m)(t_{1e}^{(m)} + Q_b)\right) \equiv e_8^{A_n};$$

$$\sigma_{23}^{A_n(1)} = \sigma_{23}^{A_n(2)} \Rightarrow$$

$$\eta_a^{A_n(m)}(-1)^m 2l_{1a}^{(m)} + \eta_b^{A_n(m)} 2\left[(-1)^m l_{1b}^{(m)}(l_{3b}^{(m)} - l_{4b}^{(m)}) + t_{1b}^{(m)}(\tilde{\Omega}_{1b}^2 - 1)\right]$$
$$+\eta_d^{A_n(m)} 2\left[(-1)^{m-1} l_{1d}^{(m)} l_{3d}^{(m)} - t_{1d}^{(m)}\right]$$
$$+\eta_e^{A_n(m)}\left((-1)^m 2l_{1e}^{(m)}[l_{2e}^{(m)} r_2^* + l_{3e}^{(m)} r_1^* - l_{4e}^{(m)} r_m^* + 2Q_b] + i(Q_b - t_{1e}^{(m)} r_m^*)\right) \equiv e_9^{A_n}. \quad (34)$$

All $e_i^{A_n}(m)$ are constant with m=1 and 2 (i.e. $e_i^{A_n}(1) = e_i^{A_n}(2)$).We seek values for $\eta_{a \text{ to } e}^{A_n(m)}$ that provide continuity at the interface for the largest number of the elastic field components $u_i^{A_n(m)}$ and $\sigma_{ij}^{A_n(m)}$. We shall proceed in a manner like that for $u_i^{(0)(m)}$ and $\sigma_{ij}^{(0)(m)}$ in Section 3.1. Restricting ourselves to terms that are singular, the trace $T_r(\sigma)^{A_n(m)}$ of the stress matrix $(\sigma)^{A_n(m)}$ takes the form

$$T_r(\sigma)^{A_n(m)}(x_1, 0, x_3) = A_n\left(t_{R1}^{A_n}(m)\frac{\partial I_0}{\partial|x_2|} + \kappa_n^2 t_{R2}^{A_n}(m)\frac{\partial \Pi_{(\pm z)}}{\partial|x_2|}\right) \quad (35)$$

With

$$t_{R1}^{A_n}(m) = \frac{1}{2}\left(e_{41}^{A_n}(m) + e_{51}^{A_n}(m) + 2e_{61}^{A_n}(m)\right),$$

$$t_{R2}^{A_n}(m) = \frac{1}{2}\left(2e_{42}^{A_n}(m) + e_{52}^{A_n}(m) - e_{62}^{A_n}(m)\right).$$

The associated oscillating part of the relative volume variation $(\Delta V/V)^{A_n(m)}$ at position $(x_1, 0, x_3)$ has the same form (35) with similar coefficients

$$V_{R1}^{A_n}(m) = \frac{(1-2\nu_m)}{2(1+\nu_m)\mu_m} t_{R1}^{A_n}(m),$$
$$V_{R2}^{A_n}(m) = \frac{(1-2\nu_m)}{2(1+\nu_m)\mu_m} t_{R2}^{A_n}(m) \, . \tag{36}$$

Several relations, interconnecting the $e_i^{A_n}(m)$ and $\eta_{a \text{ to } e}^{A_n(m)}$, are involved in the procedure under way; these are (by using (34))

$$V_{R1}^{A_n}(m) = \frac{(1-2\nu_m)}{4\nu_m\mu_m}\left\{e_{41}^{A_n}(m) - \mu_m e_1^{A_n}(m)\right\} = \frac{(1-2\nu_m)}{8(1-\nu_m)}\left\{e_1^{A_n}(m) + e_3^{A_n}(m)\right\}; \tag{a}$$

$$e_{61}^{A_n}(m) = \frac{2\nu_m\mu_m}{1-2\nu_m} V_{R1}^{A_n}(m); \tag{b}$$

$$e_{51}^{A_n}(m) = -e_{41}^{A_n}(m) + \frac{4\mu_m}{1-2\nu_m} V_{R1}^{A_n}(m) \; ; \tag{c}$$

$$e_{51}^{A_n}(m) - e_8^{A_n}(m) = \eta_e^{A_n(m)}(-1)^{m-1} 4i(2Q_c - Q_b); \tag{d}$$

$$e_9^{A_n}(m) - e_7^{A_n}(m) = \mu_m e_2^{A_n}(m) - e_7^{A_n}(m) = \eta_e^{A_n(m)} \varpi_e(m) + \eta_b^{A_n(m)} \varpi_b(m),$$

$$\varpi_e(m) = \frac{i}{2}\left(Q_b\left[r_2^*\delta_{m1} + r_1^*\delta_{m2} - 3r_m^*\right] - Q_c\left[\delta_{m1}r_2^*/\nu_2 + \delta_{m2}r_1^*/\nu_1 - 3r_m^*/\nu_m\right]\right),$$

$$\varpi_b(m) = l_{1b}^{(m)} 2(1-\tilde{\Omega}_{1b}^{1/2})\tilde{\Omega}_{1b}^{2}\left(4(1-\nu_1)(1-\nu_2)(2+\tilde{\Omega}_{1b}^{1/2})\right.$$
$$\left. - \left[2 + 3(\nu_1+\nu_2) - 4\nu_1\nu_2\right]\tilde{\Omega}_{1b}\right); \tag{e}$$

$$V_{R1}^{A_n}(m) + V_{R2}^{A_n}(m) = -\frac{1-2\nu_m}{2\mu_m}\left(\eta_b^{A_n(m)} 4\tilde{\Omega}_{1b}(1-\tilde{\Omega}_{1b})(-1)^{m-1} t_{1b}^{(m)}\right.$$
$$\left. + \eta_e^{A_n(m)} 2i(-1)^{m-1} t_{1e}^{(m)}/\nu_m\right) \; ; \tag{f}$$

$$V_{R1}^{A_n}(m) = \frac{1-2\nu_m}{2\mu_m}\Big(C_m - \eta_b^{A_n(m)}(-1)^m t_{1b}^{(m)} 4(1-\tilde{\Omega}_{1b}) + \eta_c^{A_n(m)} 4 l_{1c}^{(m)}$$
$$- \eta_d^{A_n(m)} 4(-1)^m t_{1d}^{(m)} + \eta_e^{A_n(m)} 2i(-1)^m (t_{1e}^{(m)} + Q_b)\Big). \qquad (g) \qquad (37)$$

We impose the relative volume variation to be continuous at the crossing of the interface implying that $V_{R1 \text{and } 2}^{A_n}$ (36) are constant with *m*=1 and 2. This provides values to $t_{R1 \text{and } 2}^{A_n}(m)$ that are *m*-dependent. Using (36) and (37 *a*, *b*, *c*) under $e_1^{A_n}$ also constant with *m*, we obtain successively

$$e_1^{A_n} = \frac{\mu_1\nu_1(1-2\nu_2)e_{41}^{A_n}(2) - \mu_2\nu_2(1-2\nu_1)e_{41}^{A_n}(1)}{\mu_1\mu_2(\nu_1-\nu_2)},$$

$$V_{R1}^{A_n} = \frac{(1-2\nu_1)(1-2\nu_2)\left[e_{41}^{A_n}(1)\mu_2 - e_{41}^{A_n}(2)\mu_1\right]}{4\mu_1\mu_2(\nu_1-\nu_2)},$$

$$t_{R1}^{A_n}(m) = 2\frac{\delta_{m2}\mu_2(1+\nu_2) + \delta_{m1}\mu_1(1+\nu_1)}{\delta_{m2}(1-2\nu_2) + \delta_{m1}(1-2\nu_1)} V_{R1}^{A_n},$$

$$e_{61}^{A_n}(m) = \frac{2\nu_m\left[\delta_{m2}\mu_2(1+\nu_2) + \delta_{m1}\mu_1(1+\nu_1)\right]}{(1+\nu_m)\left[\delta_{m2}(1-2\nu_2) + \delta_{m1}(1-2\nu_1)\right]} V_{R1}^{A_n},$$

$$e_{51}^{A_n}(m) = -e_{41}^{A_n}(m) + \frac{4\left[\delta_{m2}\mu_2(1+\nu_2) + \delta_{m1}\mu_1(1+\nu_1)\right]}{(1+\nu_m)\left[\delta_{m2}(1-2\nu_2) + \delta_{m1}(1-2\nu_1)\right]} V_{R1}^{A_n}. \qquad (38)$$

These expressions depend on $e_{41}^{A_n}(1)$ and $e_{41}^{A_n}(2)$; $e_1^{A_n}$ and $V_{R1}^{A_n}$ are unchanged by inverting the elastic constants, as required. The only oscillating stress that has a singularity $1/x_1$ at $(x_1, 0, x_2)$ is $\sigma_{13}^{A_n}$; hence, this stress contributes a non-zero value to the crack extension force when a non-planar interface crack (see ***Figure 1***) is subjected to general loading (this will be the subject of part III of our study). Consequently, we demand $\sigma_{13}^{A_n}$ to be continuous at the interface; this requires the associated coefficient $e_8^{A_n}$ (34) to be constant with *m*=1 and 2. Inspection of (37 *d*) tells us that taking $\eta_e^{A_n(m)} \equiv 0$ leads to $e_8^{A_n}(m) = e_{51}^{A_n}(m)$, an expression of which is displayed in (38). Imposing $e_8^{A_n}(m) = e_{51}^{A_n}(m)$ constant opens the possibility to find a relation between $e_{41}^{A_n}(1)$ and $e_{41}^{A_n}(2)$. In this way, we find

$$e_{41}^{A_n}(2) = -e_{41}^{A_n}(1)\frac{\mu_2\left[\mu_1(1-\nu_1-\nu_2) - \mu_2(1-2\nu_1)\right]}{\mu_1\left[\mu_2(1-\nu_1-\nu_2) - \mu_1(1-2\nu_2)\right]}. \qquad (39)$$

Further progression comes from (37 *e*). It tells us that first $e_9^{A_n}(m) = \mu_m e_2^{A_n}(m)$ and second $e_9^{A_n}(m) = e_7^{A_n}(m)$; to see this, it is sufficient to write down three equations corresponding successively to *m*=1 and 2, and (third) the one that results from the inversion of the elastic constants in equation for *m*=1; summing, member by member, the two last equations yields $e_9^{A_n}(2) = e_7^{A_n}(2)$ and in like manner $e_9^{A_n}(1) = e_7^{A_n}(1)$. Furthermore, we work with the value zero for $\eta_e^{A_n}$, this implies that $\eta_b^{A_n}$ must be zero. A link between $V_{R1}^{A_n}$ and $V_{R2}^{A_n}$ emerges from (37 *f*). Hence, we write

$$e_9^{A_n}(m) = e_7^{A_n}(m) = \mu_m e_2^{A_n}(m),$$
$$\eta_e^{A_n(m)} \equiv 0 \Rightarrow \eta_b^{A_n(m)} = 0,$$
$$V_{R2}^{A_n} = -V_{R1}^{A_n}. \qquad (40)$$

Under $\eta_e^{A_n}$ and $\eta_b^{A_n}$ equal to zero, we have the following relation :

$$-2\mu_m V_{R1}^{A_n}(m)/(1-2\nu_m) + \eta_d^{A_n(m)} 4\left[l_{1d}^{(m)}(l_{2d}^{(m)} + l_{3d}^{(m)}) + t_{1d}^{(m)}(-1)^m 2(1-\nu_m)\right]$$
$$= e_8^{A_n}(m) + 2(-1)^{m-1} e_7^{A_n}(m). \qquad (41)$$

It remains to find values to $e_{41}^{A_n}(1)$ and $e_7^{A_n}(m)$. This can be done by providing first a value to $\eta_d^{A_n(m)}$; for this purpose, we use (37 *g*) in which $\eta_c^{A_n(m)} \equiv 0$ ($\eta_e^{A_n}$ and $\eta_b^{A_n}$ are equally zero from (40)) and obtain

$$\eta_d^{A_n(m)} = \frac{\nu_1(1-\nu_2) + \nu_2(1-\nu_1)}{(\nu_1 C_1 - \nu_2 C_2)(1-\rho_m)}(-1)^{m-1}\left(C_m - \frac{2\mu_m}{1-2\nu_m}V_{R1}^{A_n}\right). \qquad (42)$$

Introducing this value in (41) and imposing $e_7^{A_n}(m)$ (as also $e_8^{A_n}$) to be constant with *m*=1 and 2 yields a value to $e_{41}^{A_n}(1)$ in the form

$$e_{41}^{A_n}(1) = \frac{Nu\,(e_{41}^{A_n})}{De\,(e_{41}^{A_n})} \qquad (43)$$

with

$$Nu\,(e_{41}^{A_n}) = 2\mu_1[\mu_1(1-2\nu_2) - \mu_2(1-\nu_1-\nu_2)]\{2Q_c(1-\nu_1)(1-\nu_2)(C_1 - C_2)$$
$$- Q_b[\nu_1(1-\nu_2) + \nu_2(1-\nu_1)][C_1(1-\nu_1) - C_2(1-\nu_2)]\},$$

$$De\,(e_{41}^{A_n}) = (\mu_1 - \mu_2)\{Q_c(1-\nu_1)(1-\nu_2)[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)]$$

$$- Q_b \left[\nu_1(1-\nu_2)+\nu_2(1-\nu_1)\right]\left[\mu_1(1-\nu_1)(1-2\nu_2)-\mu_2(1-\nu_2)(1-2\nu_1)\right]\} .$$

Using (43) and (39), (38) provides values

$$V_{R1}^{A_n} = \frac{Nu\ (V_{R1}^{A_n})}{De\ (V_{R1}^{A_n})} = -V_{R2}^{A_n} \qquad (44)$$

with

$$Nu\ (V_{R1}^{A_n}) = (1-2\nu_1)(1-2\nu_2)\{2Q_c(1-\nu_1)(1-\nu_2)(C_1-C_2)$$
$$- Q_b\left[\nu_1(1-\nu_2)+\nu_2(1-\nu_1)\right]\left[C_1(1-\nu_1)-C_2(1-\nu_2)\right]\},$$

$$De\ (V_{R1}^{A_n}) = 2\{Q_c(1-\nu_1)(1-\nu_2)\left[\mu_1(1-2\nu_2)-\mu_2(1-2\nu_1)\right]$$
$$- Q_b\left[\nu_1(1-\nu_2)+\nu_2(1-\nu_1)\right]\left[\mu_1(1-\nu_1)(1-2\nu_2)-\mu_2(1-\nu_2)(1-2\nu_1)\right]\};$$

$$e_8^{A_n} = e_{51}^{A_n} = \frac{Nu\ (e_8^{A_n})}{De\ (e_8^{A_n})} \qquad (45)$$

With

$$Nu\ (e_8^{A_n}) = 2\mu_1\mu_2\left[(1-2\nu_1)(1-2\nu_2)-(1-\nu_1-\nu_2)^2\right]\{2Q_c(1-\nu_1)(1-\nu_2)(C_1-C_2)$$
$$- Q_b\left[\nu_1(1-\nu_2)+\nu_2(1-\nu_1)\right]\left[C_1(1-\nu_1)-C_2(1-\nu_2)\right]\},$$

$$De\ (e_8^{A_n}) = (\nu_1-\nu_2)(\mu_1-\mu_2)\{Q_c(1-\nu_1)(1-\nu_2)\left[\mu_1(1-2\nu_2)-\mu_2(1-2\nu_1)\right]$$
$$- Q_b\left[\nu_1(1-\nu_2)+\nu_2(1-\nu_1)\right]\left[\mu_1(1-\nu_1)(1-2\nu_2)-\mu_2(1-\nu_2)(1-2\nu_1)\right]\} .$$

These values are unchanged by inverting the elastic constants, as expected. The associated $e_7^{A_n}$ and $e_2^{A_n}(m)$ are

$$e_7^{A_n} = e_9^{A_n} = \mu_m e_2^{A_n}(m) = -\frac{e_8^{A_n}}{2} + \frac{V_{R1}^{A_n}}{2}\left(\frac{\mu_1\left[C_2(-\nu_1+2\nu_1\nu_2-\nu_2^2)+\nu_2(1-\nu_1)C_1\right]}{(1-\nu_2)(1-2\nu_1)(\nu_2C_2-\nu_1C_1)}\right.$$
$$\left.+\frac{\mu_2\left[C_1(-\nu_2+2\nu_1\nu_2-\nu_1^2)+\nu_1(1-\nu_2)C_2\right]}{(1-\nu_1)(1-2\nu_2)(\nu_1C_1-\nu_2C_2)}\right) - \frac{\nu_2-\nu_1}{\nu_2C_2-\nu_1C_1}\left(\frac{C_1\left[C_2(1-2\nu_2)+C_1\right]}{4(1-\nu_2)}\right.$$
$$\left.+\frac{C_2\left[C_1(1-2\nu_1)+C_2\right]}{4(1-\nu_1)}\right) . \qquad (46)$$

Beginning with five parameters $\eta_{a\ \text{to}\ e}^{A_n(m)}$, only survive $\eta_a^{A_n(m)}$ and $\eta_d^{A_n(m)}$; the others are zero. Using (34), we have

$$\eta_a^{A_n(m)} = (-1)^m \frac{1}{2l_{1a}^{(m)}}\left(e_7^{A_n} - \eta_d^{A_n(m)} 2\left[(-1)^{m-1} l_{1d}^{(m)} l_{3d}^{(m)} - t_{1d}^{(m)}\right]\right) . \quad (47)$$

Here, $\eta_d^{A_n(m)}$ and $e_7^{A_n}$ are given by (42) and (46). With $\eta_a^{A_n(m)}$ (47) and $\eta_d^{A_n(m)}$ (42), the elastic fields $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ (32) are completely determined, thus providing the elastic fields $\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$ (31) of an interface sinusoidal climb-type edge dislocation.

IV - DISCUSSION

In the present study, we considered two welded infinitely extended elastic media $S1$ and $S2$ ***(Figure 2)***, with an interface S having the form of a corrugated sheet. We have considered a sinusoidal dislocation running indefinitely on this surface and have studied the elastic displacement and stress fields ($\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$) produced in the surrounding media. The elastic fields are decomposed individually, in the linear approximation with respect to the amplitude of the sinusoid, into two terms. The first terms correspond to the elastic fields ($\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$) of a straight dislocation on a flat interface and the second ones ($\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$) constitute an oscillating part, proportional to the sinusoid or to its spatial derivative with respect to the direction where the dislocation runs. We left with an initial vision that the elastic fields are continuous at the crossing of the interface and join those of a sinusoidal dislocation in a homogeneous infinite medium, far from the interface. In Section 3.1, expressions (28) for $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ have been obtained. The result is that in order to ensure the continuity of the elastic fields at the crossing of the interface, the relative variation of volume at the interface $(\Delta V / V)^{(0)(m)}$ exhibits a spatial dependence proportional to the delta function of Dirac $\delta(x_1)$, with a coefficient of proportionality $(-2\pi i V_R^{(0)})$ (26), well-defined, dependent on the Poisson ratios and which remains unchanged at the crossing of the interface. In addition, the following result holds

$$u_2^{(0)(m)}(x_1, \infty, x_3) = u_2^{(0)(m)\infty}(x_1, \infty, x_3) + \frac{b(1-2\nu_1)(1-2\nu_2)}{8\pi(1-2\nu_m)(1-\nu_1)(1-\nu_2)} . \quad (48)$$

In other words, under conditions of continuity at the interface, the displacement component $u_2^{(0)(m)}$ (far from the interface) differs from that of a straight dislocation $u_2^{(0)(m)\infty}$ in an infinitely extended homogeneous medium. The difference observed is constant, well defined, depends on the Poisson ratios of the two media and changes from medium *S*1 to medium *S*2. Moreover, this difference is related to the relative variation of volume at the interface, in the vicinity of dislocation. The properties which we have just described apply only when the conditions of continuity are observed. In the contrary case, for example if $V_R^{(0)} = 0$, the expression of $u_2^{(0)(m)}$ (28) is not constant with *m*=1 and 2 on the interface; as a result, $u_2^{(0)(m)}$ and $u_2^{(0)(m)\infty}$ meet far from the interface. The elastic fields $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ are completely determined in Section 3.2, thus providing the elastic fields $\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$ of an interface sinusoidal climb-type edge dislocation. One may wonder if the proposed solution is the best under condition of continuity of the elastic fields at the interface. The answer is yes because it is not possible to increase the number of elastic field components constant with *m*=1 and 2 at the interface. This is because relations interconnecting the parameters $e_i^{A_n}(m)$ exist. The relation (37*a*) shows that if $V_{R1}^{A_n}$ and $e_1^{A_n}$ are constant with *m*, then $e_{41}^{A_n}(m)$ and $e_3^{A_n}(m)$ will depend on *m*; similarly, $e_{61}^{A_n}(m)$ depends on *m* if $V_{R1}^{A_n}$ is constant with *m*, by the relation (37*b*). It will also be noted that the relation $e_9^{A_n}(m) = \mu_m e_2^{A_n}(m)$ (40) indicates that if $e_9^{A_n}$ is constant with *m* then $e_2^{A_n}(m)$ depends on *m*. In our study, the constant oscillating elastic fields with *m* at the interface are : $u_1^{A_n(m)}$, $(\Delta V/V)^{A_n(m)}$, $\sigma_{22}^{A_n(m)}$, $\sigma_{12}^{A_n(m)}$, $\sigma_{13}^{A_n(m)}$ and $\sigma_{23}^{A_n(m)}$; the others $u_2^{A_n(m)}$, $u_3^{A_n(m)}$, $\sigma_{11}^{A_n(m)}$ and $\sigma_{33}^{A_n(m)}$ are not.

V - CONCLUSION

This study considers two elastic solids *S*1 and *S*2, of infinite sizes, firmly welded along a non-planar interface *S* having the form of a corrugated sheet and provides expressions for the elastic fields (displacement and stress) of a dislocation lying indefinitely on that interface in the sinusoidal direction with a Burgers vector $\vec{b}_{II}$ ***(Figure 2)*** perpendicular to its shape (climb-type sinusoidal edge dislocation). The proposed solutions ($\vec{u}^{(m)}$, $(\sigma)^{(m)}$: Section 3) are sums of terms ($\vec{u}^{(0)(m)}$, $(\sigma)^{(0)(m)}$) corresponding to a straight interfacial edge dislocation and ($\vec{u}^{A_n(m)}$, $(\sigma)^{A_n(m)}$) proportional to the sinusoid A_n (2) or its spatial derivative $\partial A_n / \partial x_3$. It is shown that optimum continuity of the elastic fields at the crossing of the interface is achieved when the relative variation of volume is continuous at the interface and expressions of the elastic fields are written under such conditions. The forthcoming work will deal with the crack-tip stress and the crack extension force when a non-planar crack defined in Section 1 ***(Figure 1)*** is loaded in mixed mode I +II +III (work under way, part III).

REFERENCES

[1] - P. N. B. ANONGBA, *Rev. Ivoir. Sci. Technol.*, 28 (2016) 24 - 59
[2] - B. A. BILBY and J. D. ESHELBY, In : "Fracture", Ed. Academic Press (H. Liebowitz), New York, Vol. 1, (1968) 99 - 182
[3] - P. N. B. ANONGBA, *Physica Stat. Sol. B,* 194 (1996) 443 - 452
[4] - P. N. B. ANONGBA and V. VITEK, *Int. J. Fract.*, 124 (2003) 1 - 15
[5] - P. N. B. ANONGBA, *Int. J. Fract.*, 124 (2003) 17 - 31
[6] - P. N. B. ANONGBA, *Rev. Ivoir. Sci. Technol.*, 14 (2009) 55 - 86
[7] - P. N. B. ANONGBA, *Rev. Ivoir. Sci. Technol.*, 16 (2010) 11 - 50
[8] - P. N. B. ANONGBA, J. BONNEVILLE and A. JOULAIN, *Rev. Ivoir. Sci. Technol.*, 17 (2011) 37 - 53
[9] - P. N. B. ANONGBA, *Rev. Ivoir. Sci. Technol.*, 26 (2015) 76 - 90
[10] - P. N. B. ANONGBA, *Rev. Ivoir. Sci. Technol.*, 25 (2015) 34 - 55
[11] - P. N. B. ANONGBA, *Rev. Ivoir. Sci. Technol.*, 26 (2015) 36 - 75

APPENDIX A : DIFFERENCE OF THE VALUES OF $\vec{u}^{(m)\infty}$ AND $(\sigma)^{(m)\infty}$ (CLIMB-TYPE EDGE) WHEN CROSSING THE INTERFACE

Our purpose here is to write down the differences $\Delta\vec{u}^{\infty}$ and $(\Delta\sigma)^{\infty}$ (4) on crossing the interface at arbitrary point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$. We use the notation $x_2 = \xi$ ($\xi = \xi_n \sin \kappa_n x_3$ small) and take the MacLaurin series expansions of the elastic fields up to terms of first order with respect to ξ ; this means that

$$\Delta\vec{u}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta\vec{u}^{\infty}(x_1, 0, x_3) + \frac{\partial\Delta\vec{u}^{\infty}}{\partial x_2}(x_1, 0, x_3)\xi\ ,$$

$$(\Delta\sigma)^{\infty}(x_1, x_2 = \xi, x_3) = (\Delta\sigma)^{\infty}(x_1, 0, x_3) + \frac{\partial(\Delta\sigma)^{\infty}}{\partial x_2}(x_1, 0, x_3)\xi\ . \tag{A.1}$$

$\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$ are taken from our previous works [7]. We obtain (u_i is the *i*-component of vector $\vec{u}$ and σ_{ij} the *ij*-element of the stress matrix (σ) ; *i*, *j*= 1 to 3)

$$\Delta u_i^{\infty}(x_1, x_2 = \xi, x_3) = \Delta u_i^{(0)\infty} + \Delta u_i^{A_n\infty}$$

$$\Delta\sigma_{ij}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta\sigma_{ij}^{(0)\infty} + \Delta\sigma_{ij}^{A_n\infty}$$

as

$$\Delta u_1^{(0)\infty} = \frac{ibC_v}{8\pi}\int_{-\infty}^{\infty} \operatorname{sgn}(k_1) e^{ik_1x_1} dk_1\ \xi\ ,$$

$$\Delta u_1^{A_n\infty} = -A_n \frac{ibC_v}{8\pi} \int_{-\infty}^{\infty} \frac{k_1\left(k_1^2 + 2\kappa_n^2\right)}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 ;$$

$$\Delta u_2^{(0)\infty} = \frac{bC_v}{8\pi} \int_{-\infty}^{\infty} \frac{1}{|k_1|} e^{ik_1x_1} dk_1 ,$$

$$\Delta u_2^{A_n\infty} = -A_n \frac{bC_v}{8\pi} \int_{-\infty}^{\infty} \frac{3k_1^2 + 2\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1x_1} dk_1 \xi ;$$

$$\Delta u_3^{(0)\infty} = 0 ,$$

$$\Delta u_3^{A_n\infty} = -\frac{\partial A_n}{\partial x_3} \frac{bC_v}{8\pi} \int_{-\infty}^{\infty} \frac{k_1^2 + 2\kappa_n^2}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 ;$$

$$\Delta \sigma_{11}^{(0)\infty} = -6iQ_b \int_{-\infty}^{\infty} |k_1| e^{ik_1x_1} dk_1 \xi ,$$

$$\Delta \sigma_{11}^{A_n\infty} = 2iQ_b A_n \int_{-\infty}^{\infty} \frac{k_1^2 (3k_1^2 + 4\kappa_n^2)}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 ;$$

$$\Delta \sigma_{22}^{(0)\infty} = 2iQ_b \int_{-\infty}^{\infty} |k_1| e^{ik_1x_1} dk_1 \xi ,$$

$$\Delta \sigma_{22}^{A_n\infty} = -2iA_n \int_{-\infty}^{\infty} \frac{Q_b k_1^2 + 2Q_c \kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1x_1} dk_1 ;$$

$$\Delta \sigma_{33}^{(0)\infty} = -4iQ_c \int_{-\infty}^{\infty} |k_1| e^{ik_1x_1} dk_1 \xi ,$$

$$\Delta \sigma_{33}^{A_n\infty} = 4iA_n \int_{-\infty}^{\infty} \frac{k_1^2\left[2Q_c k_1^2 + \kappa_n^2(-Q_b + 4Q_c)\right] + 2Q_c \kappa_n^4}{2\left(k_1^2 + \kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 ;$$

$$\Delta \sigma_{12}^{(0)\infty} = -2Q_b \int_{-\infty}^{\infty} \mathrm{sgn}(k_1) e^{ik_1x_1} dk_1 ,$$

$$\Delta \sigma_{12}^{A_n\infty} = A_n 2Q_b \int_{-\infty}^{\infty} k_1 \frac{3k_1^2 + 2\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1x_1} dk_1 \xi ;$$

$$\Delta \sigma_{13}^{(0)\infty} = 0 ,$$

$$\Delta \sigma_{13}^{A_n\infty} = \frac{\partial A_n}{\partial x_3} 2Q_b \int_{-\infty}^{\infty} k_1 \frac{k_1^2 + 2\kappa_n^2}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 ;$$

$$\Delta \sigma_{23}^{(0)\infty} = 0 ,$$

$$\Delta \sigma_{23}^{A_n\infty} = -\frac{\partial A_n}{\partial x_3} 2i \int_{-\infty}^{\infty} \frac{(2Q_c + Q_b)k_1^2 + 2Q_c \kappa_n^2}{\left(k_1^2 + \kappa_n^2\right)^{1/2}} e^{ik_1x_1} dk_1 \xi ; \qquad \text{(A.2)}$$

where

$C_\nu = [1/(1-\nu_1) - 1/(1-\nu_2)]$, $Q_b = i(C_2 - C_1)/4$,
$Q_c = i(\nu_2 C_2 - \nu_1 C_1)/4$, $C_m = b\mu_m / 2\pi(1-\nu_m)$;

$\mathrm{sgn}(k_1) = k_1/|k_1|$; μ_m and ν_m are shear modulus and Poisson's ratio.

APPENDIX B : PARTIAL OSCILLATING ELASTIC FIELDS (CLIMB-TYPE EDGE)

The couple $(\overline{\alpha}_{3a}^{A_n(m)}, \overline{\beta}_{3a}^{A_n(m)})$ is obtained from (13 *a*, *b*, *c* and *e*) associated with the displacement. We have at position $\vec{x} = (x_1, x_2, x_3)$ $(\vec{u}^{A_n(m)V} \equiv \vec{u}_a^{A_n(m)V}$, $(\sigma)^{A_n(m)V} \equiv (\sigma)_a^{A_n(m)V}$):

$$u_{ia}^{A_n(m)V} = \frac{l_{1a}^{(m)}}{\mu_m}\left\|\delta_{i1}\frac{\partial}{\partial x_1} + \delta_{i2} + \delta_{i3}\frac{\partial}{\partial x_3}\right\| A_n\left((\delta_{i1}+\delta_{i3})\left[-\frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 I_{1a}\right]\right.$$

$$\left. -\delta_{i2}(-1)^{m-1}\left[I_0 + \kappa_n^2 \Pi_1\right]\right),$$

$$\sigma_{iia}^{A_n(m)V} = 2A_n l_{1a}^{(m)}\left((\delta_{i1}-\delta_{i2})\frac{\partial I_0}{\partial |x_2|} + (\delta_{i1}-\delta_{i3})\kappa_n^4 I_{1a} + (\delta_{i3}-\delta_{i2})\kappa_n^2\frac{\partial \Pi_1}{\partial |x_2|}\right),$$

$$\sigma_{12a}^{A_n(m)V} = 2A_n l_{1a}^{(m)}(-1)^m \frac{\partial}{\partial x_1}\left(I_0 + \kappa_n^2 \Pi_1\right),$$

$$\sigma_{j3a}^{A_n(m)V} = \frac{\partial A_n}{\partial x_3} 2 l_{1a}^{(m)}\left(\delta_{j1}\frac{\partial}{\partial x_1}\left[-\frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 I_{1a}\right] + \delta_{j2}(-1)^m\left[I_0 + \kappa_n^2 \Pi_1\right]\right); \qquad \text{(B.1)}$$

$$\Pi_z = \int_{-\infty}^{\infty}\frac{e^{(-1)^m\sqrt{k_1^2+\kappa_n^2}x_2}}{k_1^2 + z\kappa_n^2}e^{ik_1x_1}dk_1 \equiv \int_{-\infty}^{\infty}\frac{e^{-\sqrt{k_1^2+\kappa_n^2}|x_2|}}{k_1^2 + z\kappa_n^2}e^{ik_1x_1}dk_1,$$

$$I_0 = \int_{-\infty}^{\infty} e^{(-1)^m\sqrt{k_1^2+\kappa_n^2}x_2}e^{ik_1x_1}dk_1 \equiv \int_{-\infty}^{\infty} e^{-\sqrt{k_1^2+\kappa_n^2}|x_2|}e^{ik_1x_1}dk_1 = \frac{2\kappa_n|x_2|}{r}K_1,$$

$$I_{1a} = \int_{-\infty}^{\infty}\frac{e^{(-1)^m\sqrt{k_1^2+\kappa_n^2}x_2}}{(k_1^2+\kappa_n^2)^{3/2}}e^{ik_1x_1}dk_1 \equiv \int_{-\infty}^{\infty}\frac{e^{-\sqrt{k_1^2+\kappa_n^2}|x_2|}}{(k_1^2+\kappa_n^2)^{3/2}}e^{ik_1x_1}dk_1 .$$

Terms in brackets $\|\ \|$ are operators acting on A_n and others, separately; Π_1 is the value of Π_z for *z*=1, $r^2 = x_1^2 + x_2^2$ and subscripts *i*=1 to 3 and *j*=1 and 2; $K_n[x]$ is the nth-order modified Bessel function usually so denoted and δ_{ij} is the Kronecker delta. We stress that the various integrations (such as in I_{1a} and Π_z) performed in the present study are given for spatial positions satisfying the condition $(-1)^m x_2 < 0$ (*i.e.* $(-1)^{m-1} = \mathrm{sgn}(x_2)$) with $m = 1$ when $x_2 > \xi_n \sin \kappa_n x_3$ (half-space 1) and $m = 2$ when $x_2 < \xi_n \sin \kappa_n x_3$ (half-space 2). However, this makes no difference in the elastic fields to first order in ξ_n. The pair $(\overline{\alpha}_{3b}^{A_n(m)}, \overline{\beta}_{3b}^{A_n(m)})$ is obtained using (13 *a* to *d*) associated with the displacement. We have (using the similar notations)

$$u_{1b}^{A_n(m)A} = A_n \frac{l_{1b}^{(m)}}{\mu_m} \frac{\partial}{\partial x_1}\left(\kappa_n^2 l_{3b}^{(m)} I_{1a} - \frac{\partial}{\partial |x_2|}\left\{ \frac{\left[(1-\tilde{\Omega}_{1b}) l_{2b}^{(m)} + l_{4b}^{(m)}\right]\Pi_1 - l_{4b}^{(m)}\Pi_{\tilde{\Omega}_{1b}}}{1-\tilde{\Omega}_{1b}} \right\} \right),$$

$$u_{1b}^{A_n(m)B} = A_n \frac{t_{1b}^{(m)}}{\mu_m} \frac{\partial}{\partial x_1}\left((-1)^m 4(1-\nu_m) \frac{\partial}{\partial |x_2|}\left[-\Pi_1 + \tilde{\Omega}_{1b}\Pi_{\tilde{\Omega}_{1b}}\right] \right.$$

$$\left. - x_2\left[(\tilde{\Omega}_{1b} - 1) I_0 + \kappa_n^2 \Pi_1 - \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right] \right);$$

$$u_{2b}^{A_n(m)A} = \frac{A_n l_{1b}^{(m)}}{\mu_m} (-1)^m \left(l_{2b}^{(m)} I_0 + \kappa_n^2 l_{3b}^{(m)} \Pi_1 - \kappa_n^2 l_{4b}^{(m)} \Pi_{\tilde{\Omega}_{1b}} \right),$$

$$u_{2b}^{A_n(m)B} = \frac{A_n t_{1b}^{(m)}}{\mu_m} \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left((1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}} \right);$$

$$u_{3b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} \frac{l_{1b}^{(m)}}{\mu_m} \left(\kappa_n^2 l_{3b}^{(m)} I_{1a} - \frac{\partial}{\partial |x_2|}\left\{ \frac{\left[(1-\tilde{\Omega}_{1b}) l_{2b}^{(m)} + l_{4b}^{(m)}\right]\Pi_1 - l_{4b}^{(m)}\Pi_{\tilde{\Omega}_{1b}}}{1-\tilde{\Omega}_{1b}} \right\} \right),$$

$$u_{3b}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} \frac{t_{1b}^{(m)}}{\mu_m} x_2 \left[(1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right];$$

$$\sigma_{11b}^{A_n(m)A} = -2 A_n l_{1b}^{(m)} \left(-\kappa_n^4 l_{3b}^{(m)} I_{1a} + \frac{\partial}{\partial |x_2|}\left\{ -l_{2b}^{(m)} I_0 + \kappa_n^2 \left[l_{2b}^{(m)} - l_{3b}^{(m)} + \frac{l_{4b}^{(m)}}{1-\tilde{\Omega}_{1b}} \right] \Pi_1 \right.\right.$$

$$\left.\left. - \frac{\kappa_n^2 l_{4b}^{(m)} \tilde{\Omega}_{1b}}{1-\tilde{\Omega}_{1b}} \Pi_{\tilde{\Omega}_{1b}} \right\} \right),$$

$$\sigma_{11b}^{A_n(m)B} = 2 A_n t_{1b}^{(m)} \left((-1)^m 2(2-\nu_m) \frac{\partial}{\partial |x_2|}\left[(1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right] \right.$$

$$\left. - x_2 \left\{ (\tilde{\Omega}_{1b} - 1) \left\| \kappa_n^2 (1+\tilde{\Omega}_{1b}) + \frac{\partial^2}{\partial x_1^2} \right\| I_0 + \kappa_n^4 \Pi_1 - \kappa_n^4 \tilde{\Omega}_{1b}^3 \Pi_{\tilde{\Omega}_{1b}} \right\} \right);$$

$$\sigma_{22b}^{A_n(m)A} = -2A_n l_{1b}^{(m)} \frac{\partial}{\partial |x_2|}\left(l_{2b}^{(m)} I_0 + \kappa_n^2 l_{3b}^{(m)} \Pi_1 - \kappa_n^2 l_{4b}^{(m)} \Pi_{\tilde{\Omega}_{1b}}\right),$$

$$\sigma_{22b}^{A_n(m)B} = 2A_n t_{1b}^{(m)}\left((-1)^{m-1} 2(1-\nu_m)\frac{\partial}{\partial |x_2|}\left[(1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right]\right.$$

$$\left. + x_2 (1-\tilde{\Omega}_{1b})\left\{\left\| -\kappa_n^2 (1+\tilde{\Omega}_{1b}) + \frac{\partial^2}{\partial |x_2|^2}\right\| I_0 + \kappa_n^4 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right\}\right);$$

$$\sigma_{33b}^{A_n(m)A} = 2A_n l_{1b}^{(m)} \kappa_n^2 \left(\frac{1}{1-\tilde{\Omega}_{1b}} \frac{\partial}{\partial |x_2|}\left\{\left[l_{2b}^{(m)}(1-\tilde{\Omega}_{1b}) + l_{4b}^{(m)}\right]\Pi_1 - l_{4b}^{(m)} \Pi_{\tilde{\Omega}_{1b}}\right\} - \kappa_n^2 l_{3b}^{(m)} I_{1a}\right),$$

$$\sigma_{33b}^{A_n(m)B} = 2A_n t_{1b}^{(m)} \left\| -\kappa_n^2 x_2 + (-1)^m 2\nu_m \frac{\partial}{\partial |x_2|}\right\| \left((1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right);$$

$$\sigma_{12b}^{A_n(m)A} = 2A_n l_{1b}^{(m)} (-1)^m \frac{\partial}{\partial x_1}\left(l_{2b}^{(m)} I_0 + \kappa_n^2 l_{3b}^{(m)} \Pi_1 - \kappa_n^2 l_{4b}^{(m)} \Pi_{\tilde{\Omega}_{1b}}\right),$$

$$\sigma_{12b}^{A_n(m)B} = 2A_n t_{1b}^{(m)} \frac{\partial}{\partial x_1}\left(\kappa_n^2 2(1-\nu_m)\left[\Pi_1 - \tilde{\Omega}_{1b} \Pi_{\tilde{\Omega}_{1b}}\right]\right.$$

$$\left. + \left\| 3 - 2\nu_m + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|}\right\| \left\{(1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right\}\right);$$

$$\sigma_{13b}^{A_n(m)A} = 2\frac{\partial A_n}{\partial x_3} l_{1b}^{(m)} \frac{\partial}{\partial x_1}\left(\frac{\partial}{\partial |x_2|}\left\{\frac{\left[l_{2b}^{(m)}(1-\tilde{\Omega}_{1b}) + l_{4b}^{(m)}\right]\Pi_1 - l_{4b}^{(m)} \Pi_{\tilde{\Omega}_{1b}}}{\tilde{\Omega}_{1b} - 1}\right\} + \kappa_n^2 l_{3b}^{(m)} I_{1a}\right),$$

$$\sigma_{13b}^{A_n(m)B} = 2\frac{\partial A_n}{\partial x_3} t_{1b}^{(m)} \frac{\partial}{\partial x_1}\left(x_2\left[(1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right]\right.$$

$$\left. + (-1)^{m-1} 2(1-\nu_m) \frac{\partial}{\partial |x_2|}\left\{\Pi_1 - \tilde{\Omega}_{1b} \Pi_{\tilde{\Omega}_{1b}}\right\}\right);$$

$$\sigma_{23b}^{A_n(m)A} = 2\frac{\partial A_n}{\partial x_3} l_{1b}^{(m)} (-1)^m \left(l_{2b}^{(m)} I_0 + \kappa_n^2 l_{3b}^{(m)} \Pi_1 - \kappa_n^2 l_{4b}^{(m)} \Pi_{\tilde{\Omega}_{1b}}\right),$$

$$\sigma_{23b}^{A_n(m)B} = 2\frac{\partial A_n}{\partial x_3} t_{1b}^{(m)} \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|}\right\| \left\{(1-\tilde{\Omega}_{1b}) I_0 - \kappa_n^2 \Pi_1 + \kappa_n^2 \tilde{\Omega}_{1b}^2 \Pi_{\tilde{\Omega}_{1b}}\right\}; \quad \text{(B.2)}$$

The couple $(\overline{\alpha}_{1c}^{A_n(m)}, \overline{\beta}_{1c}^{A_n(m)})$ is obtained using (13 *f*, *g*, *l* and *m*) associated with stresses. We obtain

$$u_{1c}^{A_n(m)A} = -A_n \frac{l_{1c}^{(m)}}{\mu_m} \frac{\partial}{\partial x_1} \left(\kappa_n^2 I_{1a} + (1-2\nu_m) \frac{\partial}{\partial |x_2|} \Pi_1 \right),$$

$$u_{1c}^{A_n(m)B} = A_n \frac{iQ_b}{2\mu_m} \frac{\partial}{\partial x_1} \left((-1)^m 4(1-\nu_m) \frac{\partial}{\partial |x_2|} \Pi_1 + x_2 \left[\kappa_n^2 \Pi_1 - I_0 \right] \right);$$

$$u_{2c}^{A_n(m)A} = A_n \frac{l_{1c}^{(m)}}{\mu_m} (-1)^m \left((1-2\nu_m) I_0 - \kappa_n^2 \Pi_1 \right),$$

$$u_{2c}^{A_n(m)B} = A_n \frac{iQ_b}{2\mu_m} \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left(\kappa_n^2 \Pi_1 - I_0 \right);$$

$$u_{3c}^{A_n(m)A} = -\frac{\partial A_n}{\partial x_3} \frac{l_{1c}^{(m)}}{\mu_m} \left(\kappa_n^2 I_{1a} + (1-2\nu_m) \frac{\partial}{\partial |x_2|} \Pi_1 \right),$$

$$u_{3c}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} \frac{iQ_b}{2\mu_m} x_2 \left(\kappa_n^2 \Pi_1 - I_0 \right);$$

$$\sigma_{11c}^{A_n(m)A} = -2A_n l_{1c}^{(m)} \left(\kappa_n^4 I_{1a} + \frac{\partial}{\partial |x_2|} \left\{ 2(1-\nu_m)\kappa_n^2 \Pi_1 - (1-2\nu_m) I_0 \right\} \right),$$

$$\sigma_{11c}^{A_n(m)B} = A_n iQ_b \left((-1)^m 2(2-\nu_m) \frac{\partial}{\partial |x_2|} \left[\kappa_n^2 \Pi_1 - I_0 \right] \right.$$

$$\left. - x_2 \left\{ \frac{\partial^2}{\partial x_1^2} I_0 + \kappa_n^2 \left[I_0 - \kappa_n^2 \Pi_1 \right] \right\} \right);$$

$$\sigma_{22c}^{A_n(m)A} = 2A_n l_{1c}^{(m)} \frac{\partial}{\partial |x_2|} \left(\kappa_n^2 \Pi_1 - (1-2\nu_m) I_0 \right),$$

$$\sigma_{22c}^{A_n(m)B} = A_n iQ_b \left((-1)^m 2(1-\nu_m) \frac{\partial}{\partial |x_2|} \left(I_0 - \kappa_n^2 \Pi_1 \right) + x_2 \frac{\partial^2}{\partial x_1^2} I_0 \right);$$

$$\sigma_{33c}^{A_n(m)A} = 2A_n l_{1c}^{(m)} \kappa_n^2 \left(\kappa_n^2 I_{1a} + (1-2\nu_m) \frac{\partial}{\partial |x_2|} \Pi_1 \right),$$

$$\sigma_{33c}^{A_n(m)B} = A_n iQ_b \left\| \kappa_n^2 x_2 + (-1)^{m-1} 2\nu_m \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - \kappa_n^2 \Pi_1 \right);$$

$$\sigma_{12c}^{A_n(m)A} = 2A_n l_{1c}^{(m)} (-1)^m \frac{\partial}{\partial x_1} \left((1-2\nu_m) I_0 - \kappa_n^2 \Pi_1 \right),$$

$$\sigma_{12c}^{A_n(m)B} = -A_n iQ_b \frac{\partial}{\partial x_1}\left(\kappa_n^2 2(1-\nu_m)\Pi_1 + \left\| 3 - 2\nu_m + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \{I_0 - \kappa_n^2 \Pi_1\} \right);$$

$$\sigma_{13c}^{A_n(m)A} = -\frac{\partial A_n}{\partial x_3} 2 l_{1c}^{(m)} \frac{\partial}{\partial x_1}\left((1-2\nu_m)\frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 I_{1a} \right),$$

$$\sigma_{13c}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} iQ_b \frac{\partial}{\partial x_1}\left((-1)^m 2(1-\nu_m)\frac{\partial \Pi_1}{\partial |x_2|} - x_2\{I_0 - \kappa_n^2 \Pi_1\} \right);$$

$$\sigma_{23c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2 l_{1c}^{(m)} (-1)^m \left((1-2\nu_m) I_0 - \kappa_n^2 \Pi_1 \right),$$

$$\sigma_{23c}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3} iQ_b \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - \kappa_n^2 \Pi_1 \right); \qquad \text{(B.3)}$$

The pair $(\overline{\alpha}_{1d}^{A_n(m)}, \overline{\beta}_{1d}^{A_n(m)})$ is calculated from (13 *h*, *i* and *o*, *p*) associated with stresses. We have

$$u_{1d}^{A_n(m)A} = -A_n \frac{l_{1d}^{(m)}}{\mu_m} \frac{\partial}{\partial x_1}\left(l_{2d}^{(m)} \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 l_{3d}^{(m)} I_{1a} \right),$$

$$u_{1d}^{A_n(m)B} = A_n \frac{t_{1d}^{(m)}}{\mu_m} \frac{\partial}{\partial x_1}\left((-1)^{m-1} 4(1-\nu_m)\frac{\partial \Pi_1}{\partial |x_2|} + x_2\{I_0 - \kappa_n^2 \Pi_1\} \right);$$

$$u_{2d}^{A_n(m)A} = A_n \frac{l_{1d}^{(m)}}{\mu_m} (-1)^m \left(l_{2d}^{(m)} I_0 - \kappa_n^2 l_{3d}^{(m)} \Pi_1 \right),$$

$$u_{2d}^{A_n(m)B} = A_n \frac{t_{1d}^{(m)}}{\mu_m} \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - \kappa_n^2 \Pi_1 \right);$$

$$u_{3d}^{A_n(m)A} = -\frac{\partial A_n}{\partial x_3} \frac{l_{1d}^{(m)}}{\mu_m} \left(l_{2d}^{(m)} \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 l_{3d}^{(m)} I_{1a} \right),$$

$$u_{3d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} \frac{t_{1d}^{(m)}}{\mu_m} x_2 \left(I_0 - \kappa_n^2 \Pi_1 \right);$$

$$\sigma_{11d}^{A_n(m)A} = A_n 2 l_{1d}^{(m)} \left(\frac{\partial}{\partial |x_2|}\left[l_{2d}^{(m)} I_0 - \kappa_n^2 (l_{2d}^{(m)} + l_{3d}^{(m)})\Pi_1 \right] - \kappa_n^4 l_{3d}^{(m)} I_{1a} \right),$$

$$\sigma_{11d}^{A_n(m)B} = A_n 2 t_{1d}^{(m)} \left((-1)^m 2(2-\nu_m) \frac{\partial}{\partial |x_2|}\left(I_0 - \kappa_n^2 \Pi_1 \right) + x_2 \left\{ \left\| \kappa_n^2 + \frac{\partial^2}{\partial x_1^2} \right\| I_0 - \kappa_n^4 \Pi_1 \right\} \right);$$

$$\sigma_{22d}^{A_n(m)A} = -A_n 2 l_{1d}^{(m)} \frac{\partial}{\partial |x_2|}\left(l_{2d}^{(m)} I_0 - \kappa_n^2 l_{3d}^{(m)} \Pi_1 \right),$$

$$\sigma_{22d}^{A_n(m)B} = A_n 2 t_{1d}^{(m)} \left((-1)^{m-1} 2(1-\nu_m) \frac{\partial}{\partial |x_2|}\left(I_0 - \kappa_n^2 \Pi_1 \right) - x_2 \frac{\partial^2 I_0}{\partial x_1^2} \right);$$

$$\sigma_{33d}^{A_n(m)A} = A_n 2 l_{1d}^{(m)} \kappa_n^2 \left(l_{2d}^{(m)} \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 l_{3d}^{(m)} I_{1a} \right),$$

$$\sigma_{33d}^{A_n(m)B} = -A_n 2 t_{1d}^{(m)} \left\| \kappa_n^2 x_2 + (-1)^{m-1} 2\nu_m \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - \kappa_n^2 \Pi_1 \right);$$

$$\sigma_{12d}^{A_n(m)A} = A_n 2 l_{1d}^{(m)} (-1)^m \frac{\partial}{\partial x_1}\left(l_{2d}^{(m)} I_0 - \kappa_n^2 l_{3d}^{(m)} \Pi_1 \right),$$

$$\sigma_{12d}^{A_n(m)B} = A_n 2 t_{1d}^{(m)} \frac{\partial}{\partial x_1}\left(\kappa_n^2 2(1-\nu_m)\Pi_1 + \left\| 3 - 2\nu_m + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - \kappa_n^2 \Pi_1 \right) \right);$$

$$\sigma_{13d}^{A_n(m)A} = -\frac{\partial A_n}{\partial x_3} 2 l_{1d}^{(m)} \frac{\partial}{\partial x_1}\left(l_{2d}^{(m)} \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 l_{3d}^{(m)} I_{1a} \right),$$

$$\sigma_{13d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2 t_{1d}^{(m)} \frac{\partial}{\partial x_1}\left((-1)^{m-1} 2(1-\nu_m) \frac{\partial \Pi_1}{\partial |x_2|} + x_2 \left\{ I_0 - \kappa_n^2 \Pi_1 \right\} \right);$$

$$\sigma_{23d}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2 l_{1d}^{(m)} (-1)^m \left(l_{2d}^{(m)} I_0 - \kappa_n^2 l_{3d}^{(m)} \Pi_1 \right),$$

$$\sigma_{23d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2 t_{1d}^{(m)} \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - \kappa_n^2 \Pi_1 \right); \qquad \text{(B.4)}$$

The couple $(\overline{\alpha}_{3e}^{A_n(m)}, \overline{\beta}_{3e}^{A_n(m)})$ is obtained from (13 j, k, and n, l) associated with stresses. We have (here, we write r_m for r_m^*, m=1 and 2)

$$u_{1e}^{A_n(m)A} = A_n \frac{l_{1e}^{(m)}}{\mu_m} \frac{\partial}{\partial x_1}\left(-\frac{l_{2e}^{(m)} r_2}{1+r_2} \frac{\partial \Pi_{(-r_2)}}{\partial |x_2|} - \frac{l_{3e}^{(m)} r_1}{1+r_1} \frac{\partial \Pi_{(-r_1)}}{\partial |x_2|} + \frac{l_{4e}^{(m)} r_m}{1+r_m} \frac{\partial \Pi_{(-r_m)}}{\partial |x_2|} \right.$$

$$\left. - \left[\frac{l_{2e}^{(m)}}{1+r_2} + \frac{l_{3e}^{(m)}}{1+r_1} - \frac{l_{4e}^{(m)}}{1+r_m} - 2Q_b + 4Q_c \right] \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 2 Q_b I_{1a} \right),$$

$$u_{1e}^{A_n(m)B} = -\frac{A_n i}{2\mu_m} \frac{\partial}{\partial x_1}\left(t_{1e}^{(m)} \left\| \kappa_n^2 r_m x_2 + 4(1-\nu_m)(-1)^{m-1} \frac{\partial}{\partial |x_2|} \right\| \Pi_{(-r_m)} \right.$$

$$+ x_2 (t_{1e}^{(m)} + Q_b) I_0 - t_{2e}^{(m)} \left\| \kappa_n^2 x_2 + 4(1-\nu_m)(-1)^m \frac{\partial}{\partial |x_2|} \right\| \Pi_1 \Bigg);$$

$$u_{2e}^{A_n(m)A} = \frac{A_n l_{1e}^{(m)}}{\mu_m} (-1)^m \left(\kappa_n^2 \left[2Q_b \Pi_1 + l_{2e}^{(m)} r_2 \Pi_{(-r_2)} + l_{3e}^{(m)} r_1 \Pi_{(-r_1)} - l_{4e}^{(m)} r_m \Pi_{(-r_m)} \right] \right.$$

$$\left. + \left[l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} - 2Q_b + 4Q_c \right] I_0 \right),$$

$$u_{2e}^{A_n(m)B} = -\frac{A_n i}{2\mu_m} \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| \left((t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right);$$

$$u_{3e}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} \frac{l_{1e}^{(m)}}{\mu_m} \left(-\frac{l_{2e}^{(m)} r_2}{1+r_2} \frac{\partial \Pi_{(-r_2)}}{\partial |x_2|} - \frac{l_{3e}^{(m)} r_1}{1+r_1} \frac{\partial \Pi_{(-r_1)}}{\partial |x_2|} + \frac{l_{4e}^{(m)} r_m}{1+r_m} \frac{\partial \Pi_{(-r_m)}}{\partial |x_2|} \right.$$

$$\left. - \left[\frac{l_{2e}^{(m)}}{1+r_2} + \frac{l_{3e}^{(m)}}{1+r_1} - \frac{l_{4e}^{(m)}}{1+r_m} - 2Q_b + 4Q_c \right] \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^2 2Q_b I_{1a} \right),$$

$$u_{3e}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3} \frac{i}{2\mu_m} x_2 \left((t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right);$$

$$\sigma_{11e}^{A_n(m)A} = 2A_n l_{1e}^{(m)} \left(\kappa_n^2 \left\{ \frac{l_{2e}^{(m)} r_2^2}{1+r_2} \frac{\partial \Pi_{(-r_2)}}{\partial |x_2|} + \frac{l_{3e}^{(m)} r_1^2}{1+r_1} \frac{\partial \Pi_{(-r_1)}}{\partial |x_2|} - \frac{l_{4e}^{(m)} r_m^2}{1+r_m} \frac{\partial \Pi_{(-r_m)}}{\partial |x_2|} \right\} \right.$$

$$- \kappa_n^2 \left[-\frac{l_{2e}^{(m)} r_2}{1+r_2} - \frac{l_{3e}^{(m)} r_1}{1+r_1} + \frac{l_{4e}^{(m)} r_m}{1+r_m} - 2Q_b \right] \frac{\partial \Pi_1}{\partial |x_2|} + \kappa_n^4 2Q_b I_{1a}$$

$$\left. + \left(l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} - 2Q_b + 4Q_c \right) \frac{\partial}{\partial |x_2|} \left\{ I_0 - \kappa_n^2 \Pi_1 \right\} \right),$$

$$\sigma_{11e}^{A_n(m)B} = A_n i \left\| (-1)^{m-1} 2(2-\nu_m) \frac{\partial}{\partial |x_2|} - x_2 \frac{\partial^2}{\partial x_1^2} \right\|$$

$$\times \left((t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right);$$

$$\sigma_{22e}^{A_n(m)A} = -2A_n l_{1e}^{(m)} \frac{\partial}{\partial |x_2|} \left(\kappa_n^2 \left[2Q_b \Pi_1 + l_{2e}^{(m)} r_2 \Pi_{(-r_2)} + l_{3e}^{(m)} r_1 \Pi_{(-r_1)} - l_{4e}^{(m)} r_m \Pi_{(-r_m)} \right] \right.$$

$$\left. + \left[l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} - 2Q_b + 4Q_c \right] I_0 \right),$$

$$\sigma_{22e}^{A_n(m)B} = A_n i \left\| (-1)^m 2(1-\nu_m) \frac{\partial}{\partial |x_2|} - x_2 \frac{\partial^2}{\partial |x_2|^2} \right\|$$

$$\times \left((t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right);$$

$$\sigma_{33e}^{A_n(m)A} = 2A_n l_{1e}^{(m)} \kappa_n^2 \left(\frac{l_{2e}^{(m)} r_2}{1+r_2} \frac{\partial \Pi_{(-r_2)}}{\partial |x_2|} + \frac{l_{3e}^{(m)} r_1}{1+r_1} \frac{\partial \Pi_{(-r_1)}}{\partial |x_2|} - \frac{l_{4e}^{(m)} r_m}{1+r_m} \frac{\partial \Pi_{(-r_m)}}{\partial |x_2|} \right.$$

$$\left. + \left[\frac{l_{2e}^{(m)}}{1+r_2} + \frac{l_{3e}^{(m)}}{1+r_1} - \frac{l_{4e}^{(m)}}{1+r_m} - 2Q_b + 4Q_c \right] \frac{\partial \Pi_1}{\partial |x_2|} - \kappa_n^2 2Q_b I_{1a} \right) ,$$

$$\sigma_{33e}^{A_n(m)B} = A_n i \left\| (-1)^{m-1} 2\nu_m \frac{\partial}{\partial |x_2|} + x_2 \kappa_n^2 \right\|$$

$$\times \left((t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right);$$

$$\sigma_{12e}^{A_n(m)A} = 2A_n l_{1e}^{(m)} (-1)^m \frac{\partial}{\partial x_1} \left(\kappa_n^2 \left[2Q_b \Pi_1 + l_{2e}^{(m)} r_2 \Pi_{(-r_2)} + l_{3e}^{(m)} r_1 \Pi_{(-r_1)} - l_{4e}^{(m)} r_m \Pi_{(-r_m)} \right] \right.$$

$$\left. + \left[l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} - 2Q_b + 4Q_c \right] I_0 \right) ,$$

$$\sigma_{12e}^{A_n(m)B} = -A_n i \frac{\partial}{\partial x_1} \left(\kappa_n^2 2(1-\nu_m) \left(t_{1e}^{(m)} \Pi_{(-r_m)} + Q_b \Pi_1 \right) + \left\| 3 - 2\nu_m + x_2 (-1)^{m-1} \frac{\partial}{\partial |x_2|} \right\| \right.$$

$$\left. \times \left\{ (t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right\} \right);$$

$$\sigma_{13e}^{A_n(m)A} = -\frac{\partial A_n}{\partial x_3} 2 l_{1e}^{(m)} \frac{\partial}{\partial x_1} \left(\frac{l_{2e}^{(m)} r_2}{1+r_2} \frac{\partial \Pi_{(-r_2)}}{\partial |x_2|} + \frac{l_{3e}^{(m)} r_1}{1+r_1} \frac{\partial \Pi_{(-r_1)}}{\partial |x_2|} - \frac{l_{4e}^{(m)} r_m}{1+r_m} \frac{\partial \Pi_{(-r_m)}}{\partial |x_2|} \right.$$

$$\left. + \left[\frac{l_{2e}^{(m)}}{1+r_2} + \frac{l_{3e}^{(m)}}{1+r_1} - \frac{l_{4e}^{(m)}}{1+r_m} - 2Q_b + 4Q_c \right] \frac{\partial \Pi_1}{\partial |x_2|} - \kappa_n^2 2Q_b I_{1a} \right) ,$$

$$\sigma_{13e}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} i \frac{\partial}{\partial x_1} \left((-1)^m 2(1-\nu_m) \frac{\partial}{\partial |x_2|} \left(t_{1e}^{(m)} \Pi_{(-r_m)} + Q_b \Pi_1 \right) \right.$$

$$\left. - x_2 \left\{ (t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right\} \right);$$

$$\sigma_{23e}^{A_n(m)A} = \frac{\partial A_n}{\partial x_1} 2 l_{1e}^{(m)} (-1)^m \left(\kappa_n^2 \left[2Q_b \Pi_1 + l_{2e}^{(m)} r_2 \Pi_{(-r_2)} + l_{3e}^{(m)} r_1 \Pi_{(-r_1)} - l_{4e}^{(m)} r_m \Pi_{(-r_m)} \right] \right.$$

$$\left. + \left[l_{2e}^{(m)} + l_{3e}^{(m)} - l_{4e}^{(m)} - 2Q_b + 4Q_c \right] I_0 \right) ,$$

$$\sigma_{23e}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} i \left\| (-1)^m x_2 \frac{\partial}{\partial |x_2|} - 1 \right\| \left((t_{1e}^{(m)} + Q_b) I_0 + \kappa_n^2 \left[t_{1e}^{(m)} r_m \Pi_{(-r_m)} - Q_b \Pi_1 \right] \right); \quad \text{(B.5)}$$

$$\Pi_{(-z)} \equiv \int_{-\infty}^{\infty} \frac{e^{-\sqrt{k_1^2 + \kappa_n^2}\,|x_2|}}{k_1^2 - z\kappa_n^2} e^{ik_1 x_1} dk_1 .$$