ISSN 1813-3290, http://www.revist.ci

STRESS CONCENTRATION AND SHAPE OPTIMIZATION ON A PERFORATED PLATE AND A SHOULDERED PLATE

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ABSTRACT

The aim of this study was to show that the problem of weakening a structure as the result of local excess stress can be solved by a modification of the geometric shape of the weakened area. To address this, it is necessary to know the stress concentration factor associated to each hole shape. The concept is hereby discussed to propose the best shape which meets the criterion of resistance during the design phase. The case of the traction of a breakthrough plate with a hole is detailly analyzed before other hubs discussions such as the shoulders. The strategy will support the numerical analysis code COMSOL of the field stress at the level of singularities.

Keywords : *optimization, plate, stress, shape, weakening.*

RÉSUMÉ

Concentration de contraintes et optimisation de forme d'une plaque trouée et une plaque épaulée

Ce travail a pour objectif de montrer que le problème d'affaiblissement d'une structure résultant des contraintes locales peut être résolu par une modification de la forme géométrique de la zone fragilisée. Pour ce faire, il est nécessaire de connaître le facteur de concentration de contrainte associé à chaque géométrie, notion abordée dans ce travail, afin de proposer une forme répondant au mieux au critère de résistance pendant la phase de conception. Le cas de la traction d'une plaque percée d'un trou est analysé dans le détail avant d'évoquer d'autres concentrateurs tels que les épaulements. La stratégie s'appuie sur l'analyse numérique par le code COMSOL du champ de contrainte au niveau des singularités.

Mots-clés : *optimisation, plaque, contraintes, formes, affaiblissement.*

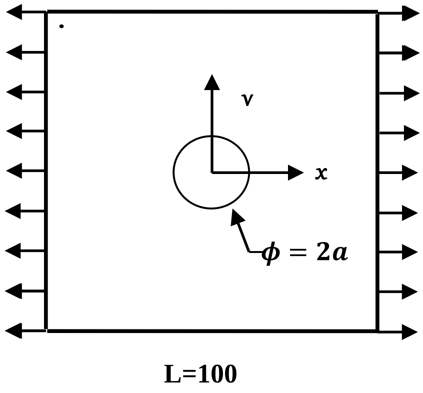
I - INTRODUCTION

Field design still represents to an engineer a complicated and laborious task because the most efficient research area is difficult to obtain, as well as the most realistic design which fully satisfies the requirements imposed upon it. [1-3].The predictability of numerical models associated to increased efficiency of technological information now allows the consideration of more complex and numerous analyses in the design phase [4]. In the field of bolted or riveted assemblies and swivel mechanisms, the components which are generally used present holes at bindings level ,nicks or shoulders, seat of stress concentration [5-7]. This work emphasizes on the reduction of these excess stresses in two applications: a breakthrough square plate with a hole in its center, and a square with flange at each end plate. The optimization setting is focused on the shape of the hole in the first case and the radius of the shoulder for the second. The aim is the minimization of the maximum value of the equivalent Von Mises stress at the level of the singularities [8-9].

II - MATERIAL AND METHODS

II-1. A breakthrough plate with a hole in traction

A square plate of thickness h breakthrough of a cylindrical hole circular cross section of radius a is subject to its end to a state of simple traction. The geometry and the problem details are illustrated by the diagram in ***Figure 1***. Using the symmetries of geometry and loading, the axis of the hole is Oy and the direction of traction is Ox. The side plate L is assumed large enough compared to a so that stress state far from the hole is not affected by the presence of the hole and can therefore be assimilated to the following homogeneous state: $\sigma^{\infty} = P$
Where $P > 0$ is the loading parameter of the structure. The edge of the hole as well as the $y = \mp L$ areas are free of efforts.



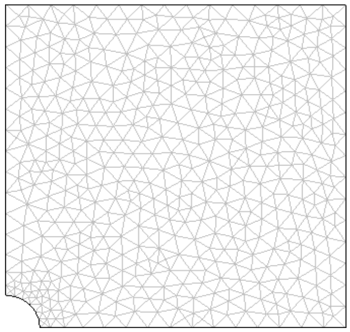


Figure 1 : *Geometry of the reference problem (a), mesh (b)*

A system of polar coordinates $(\mathrm{O}, \mathrm{r}, \theta)$ is adopted.
O is the center of the hole and the angle θ measured from Ox. Considering loading, the problem is treated in linear isotropic elasticity, in the case of plane stress. The field of stress is found in the form:

$$\sigma_{rr}(r,\theta), \sigma_{\theta\theta}(r,\theta), \sigma_{r\theta}(r,\theta), \sigma_{ZZ} = \sigma_{rZ} = \sigma_{\theta Z} = 0 \qquad (1)$$

The reference solution is written as [10-11]:

$$\sigma_{rr}(r,\theta) = \frac{P}{2}\left[\left(1 - \left(\frac{a}{r}\right)^2\right) - \left(1 - 4.\left(\frac{a}{r}\right)^2 + 3.\left(\frac{a}{r}\right)^4\right).cos2\theta\right] \qquad (2)$$

$$\sigma_{\theta\theta}(r,\theta) = \frac{P}{2}\left[\left(1 + \left(\frac{a}{r}\right)^2\right) + \left(1 + 3.\left(\frac{a}{r}\right)^4\right).cos2\theta\right] \qquad (3)$$

$$\sigma_{r\theta}(r,\theta) = \frac{P}{2}\left[\left(1 + 2.\left(\frac{a}{r}\right)^2 - 3.\left(\frac{a}{r}\right)^4\right).sin2\theta\right] \qquad (4)$$

In particular, at the edge of the hole (r=a), we have:

$$\sigma_{\theta\theta} = P[1 + 2.cos2\theta] \qquad (5)$$

and along the x axis:

$$\sigma_{\theta\theta} = \sigma_{yy} = \frac{P}{2}\left[\left(1 + \left(\frac{a}{r}\right)^2\right) + \left(1 + 3.\left(\frac{a}{r}\right)^4\right)\right] \qquad (7)$$

This field of stress indicates that stress is not homogeneous in a breakthrough plate sought in traction at its end [12]. The rapid decrease in $1/r^2$of the field of stress ensures that these heterogeneities grow only in the vicinity of the hole, and the field, sufficiently far from the hole can be considered as homogeneous. Thus, there is excess of stress in edge of the hole and called: stress concentration they have an essential role in initiating the rupture. The structure is modeled by finite element method, by 994 three-node triangular elements. The material is steel's supposed linear elastic behavior. Material characteristics are given in the ***Table 1*** below:

Table 1 : *Mechanical characteristics of the material of the plate*

Matérial	**E** (*MPa*)	**ν**	**Rc** (*MPa*)
Steel	210 000	0.3	250000

II-2. The plate with flange in compression

The absence of material or hole is not the only morphological peculiarity which may create a stress riser. Generally, the notches, the taps shoulders and another leave of connection lead to weakening of the structure. It is proposed to study the influence of the leave on the localized excess of stress of the shoulder, to the end of the plate. The 2D plate model is represented in ***Figure 4***. It has a numerically modeling with COMSOL. The movements of the lower area are blocked and the shoulders end support vertical pressure $F=10^5 Pa/m^2$. The material, the dimensions of the plate and the refinement of mesh are the same a those of the previous study.

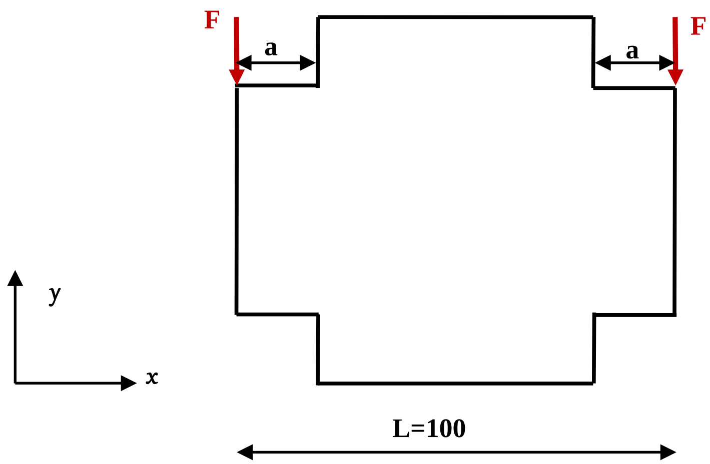


Figure 2 : *Geometry of plate with flange in compression*

III - RESULTS AND DISCUSSION

The calculation is performed digitally by the COMSOL finite element code. ***Figure 3*** shows the distribution of the stress traction. The areas of stress concentration at the edge of the hole are well distinguished. We associated the Kt, stress concentration coefficient, greater than 1, which is defined as the ratio of the real maximum stress in discontinuity area to the stress in the exact section.

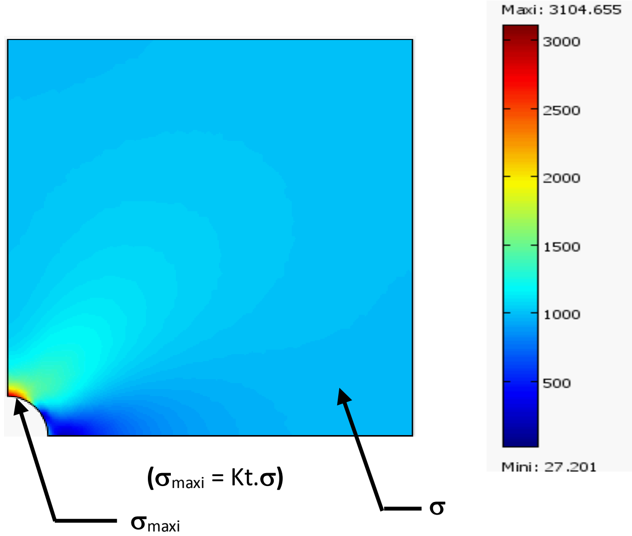


Figure 3 : *Equivalent stress distribution of Von Mises on a plate breakthrough with a circular hole in traction*

It intends subsequently to study the influence of the shape of the hole on the coefficient Kt. We calculated the maximum stress for the main geometry of the hole. The area occupied by the hole is the same in each case and the basic model used is the same (material and solicitations), as the level of refinement of mesh.

The different outcomes are summarized in the ***Table 2*** below that displays the maximum stress and strain Kt concentration factor in each case.

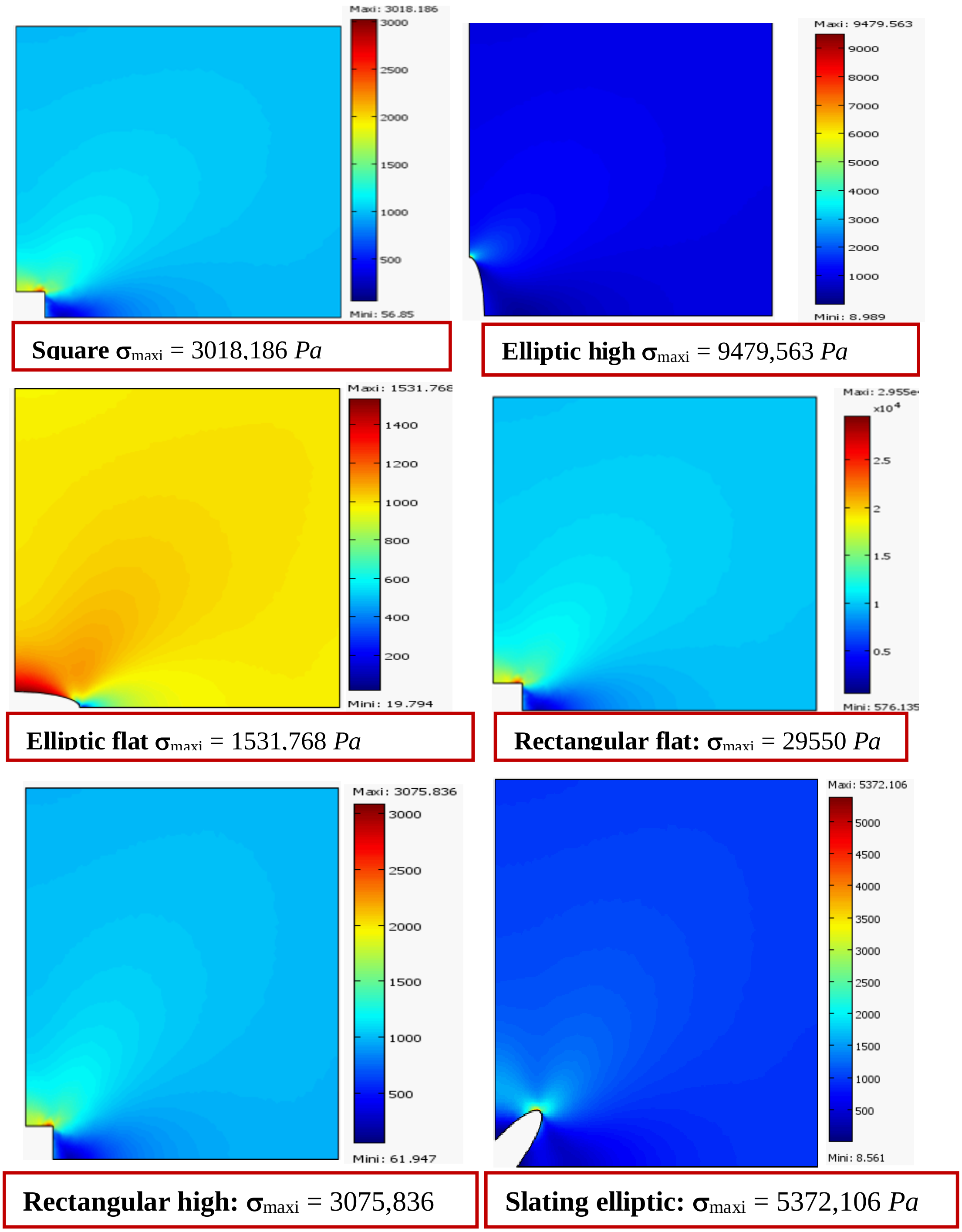
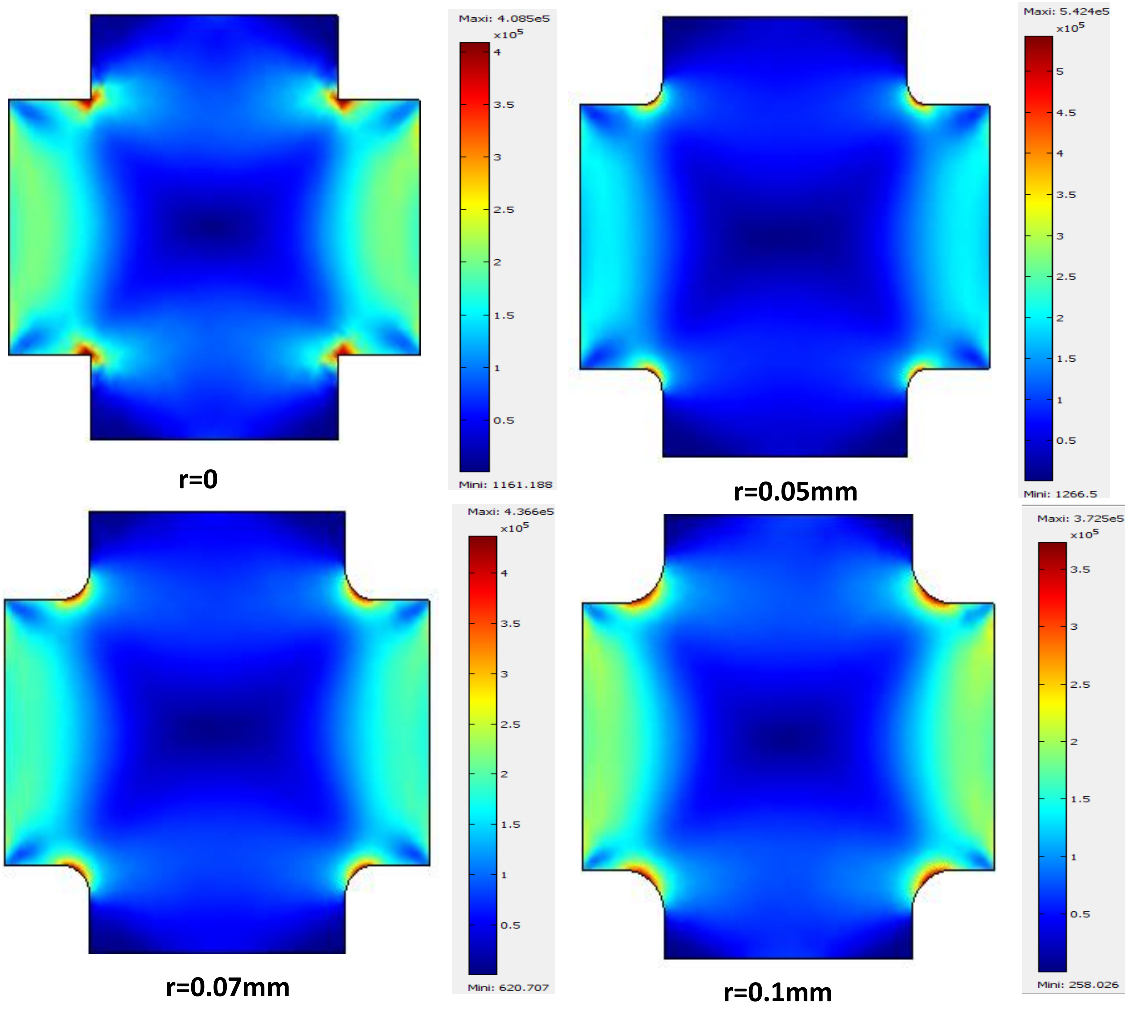


Figure 4: *The Von Mises equivalent field of stress on a plate breakthrough in traction*

Table 2 : *Stress concentration factor for some geometries of the hole on a plate in traction*

Geometry of a hole	Square	Rectangular		Elliptical			Circular
		flat	high	high	flat	slating	
Von Misesequivalent stress $\sigma_{\max}(Pa)$	3018,186	29550	3075,836	9479,563	1531,768	5372,106	3000,104
Concentration factor Of stress	3,018	29,550	3,075	9,479	1,531	5,372	3

We also calculated the maximum stress for a series of leave radius between 0 and 0.2 mm. ***Figure 5*** shows the Von Mises equivalent stress distribution in the plate for each case.



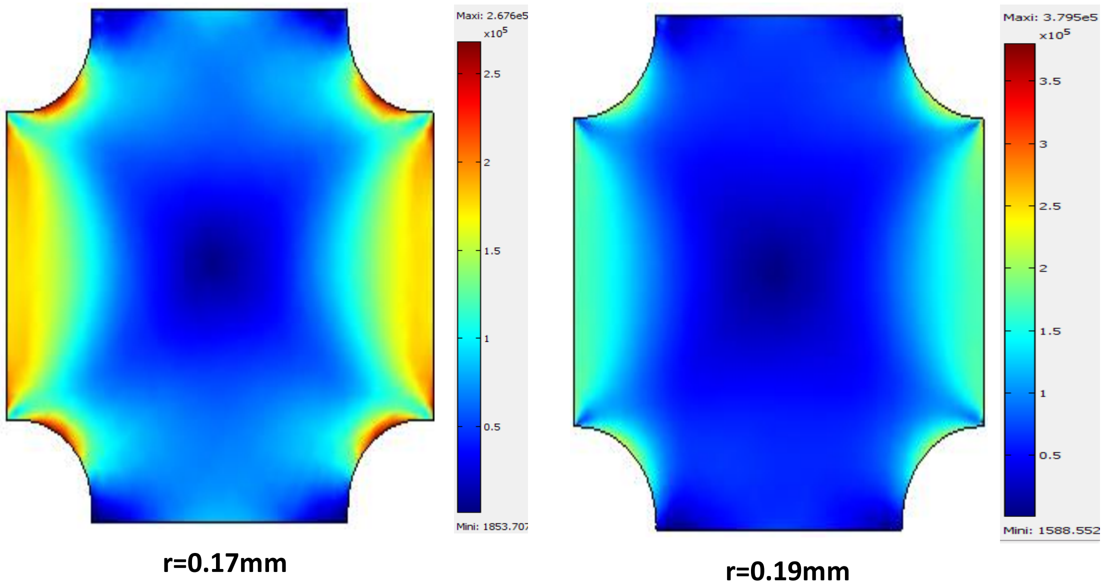


Figure 5 : *Distribution of the Von Mises equivalent stress in the mid-body plate*

Figure 6 shows the evolution of the maximum stress in the plate, depending on the fillet radius

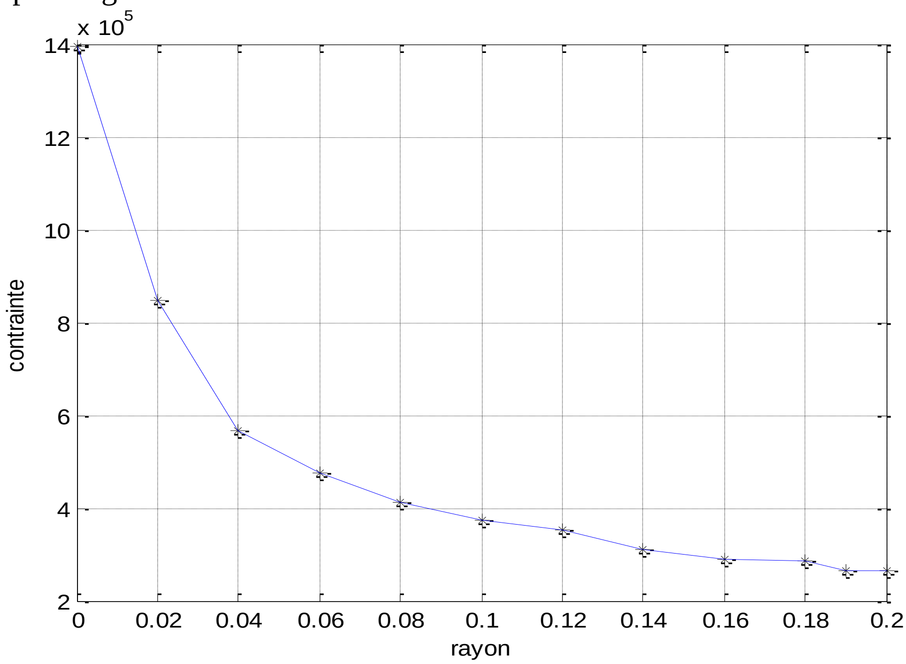


Figure 6 : *Influence of the leave radius on the maximum stress*

This curve shows that the value of the maximum stress quickly decreases when the fillet radius increases. This until we reach a limit value that materialized on the curve by an asymptote. This result reflects the fact that the increased radius above at a certain value has more bearing on the stress.

IV - CONCLUSION

This work has shown that the presence of holes, and more generally notches and other fillet radius leads to a weakening of the structure, due to local excess stresses, called stress concentrations. Therefore, it would be wise to avoid as much as possible drilling or machining defects or functional parts of this type. When the presence of stress is unavoidable, it is necessary to know the stress concentration factor associate to each geometry, in order to choose the form that guarantees the proper functioning of the piece. The case of the traction of breakthrough plate with a hole favors elliptical holes facing parallel to traction load to reduce stress around the hole.

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