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NON-PLANAR INTERFACE CRACK UNDER GENERAL LOADING I. *GLIDE-TYPE EDGE AND SCREW SINUSOIDAL DISLOCATIONS*

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ABSTRACT

This study's objective is to analyze the conditions of propagation of an oscillatory front crack along a non-planar interface, under mixed mode I + II + III loading. The crack model consists of a continuous distribution of three families of non-straight dislocations having the shape of the crack front: families 1 and 2 are edges (on average) and family 3 is screw. The associated Burgers vectors $\vec{b}_j$ (j= I, II, III) are directed along the applied tension and shears x_2, x_1 and x_3 directions, respectively. The dislocations are aligned along the x_3-direction and spread in x_2x_3-planes in a small oscillating form $\xi(x_1, x_3)$ at an average elevation $h(x_1)$. In this part I of the study, the displacement and stress fields of dislocations with $\vec{b}_I$ and $\vec{b}_{III}$ are given. Results are displayed, that make easily accessible stress terms with singularities $1/x_1$ and $\delta(x_1)$ (Dirac delta function), involved in the crack analysis to come (Part II of this work).

Keywords : *linear elasticity, interface dislocations, Galerkin vector, three-dimensional biharmonic functions, Fourier forms, linear systems of equations.*

RÉSUMÉ

Fissure d'interface non plane sous sollicitation extérieure arbitraire I. *Dislocations vis et coins de type glissile*

La présente étude se fixe pour objectif d'analyser les conditions de propagation d'une fissure de front oscillatoire le long d'une interface non plane sous sollicitation en mode mixte I+II+III. Le modèle de fissure adopté, est une distribution continue de trois familles de dislocations non rectilignes ayant la forme du front de fissure : les familles 1 et 2 sont des coins (en moyenne) et la famille 3 est vis. Les vecteurs de Burgers infinitésimaux associés $\vec{b}_j$ (j= I, II, III) sont suivant les directions x_2, x_1 and x_3, correspondant à la tension et aux cisaillements appliqués, respectivement. Les dislocations sont suivant la direction x_3 et s'étalent dans les plans x_2x_3 dans la forme $\xi(x_1, x_3)$ à la hauteur $h(x_1)$. Dans cette partie I de l'étude, les champs de contrainte et de déplacement des dislocations de vecteurs de Burgers $\vec{b}_I$ et $\vec{b}_{III}$ sont donnés. Les résultats sont présentés de façon à rendre facilement accessibles les termes de contrainte avec les singularités $1/x_1$ et $\delta(x_1)$ (fonction delta de Dirac) impliquées dans l'analyse des fissures (partie II) à venir de notre travail.

Mots-clés : *élasticité linéaire, dislocations d'interface, Vecteur de Galerkin, fonctions biharmoniques à trois dimensions, expansions en séries de Fourier, systèmes d'équations linéaires.*

I - INTRODUCTION

The main objective of this study is to analyze the conditions for the propagation of a crack, along a non-planar interface R of arbitrary shape, in a pair of two firmly welded different solids $R1$ and $R2$. This work is fundamental by the fact that most of the materials, used in practice, are composite materials that can deteriorate in service by the propagation of interface cracks. To a large crack under general applied loading, fluctuating about average fracture plane (e.g. Ox_1x_3 of a Cartesian coordinate system x_i) normal to the applied tension direction, the following description applies locally : x_1, average crack propagation direction in that plane; x_2x_3, local plane of the crack front and x_3, average crack-front direction. Hence, we can define a simple model by specifying that the crack extends from $x_1 = -a$ to a, with a front lying in the x_2x_3 – plane in a general form $x_2 = f(x_1, x_3)$ and an average direction that runs indefinitely along x_3. This is the model of interface crack ***(Figure 1)*** that we shall adopt in the present study where $R1$ and $R2$ are confined for illustration purpose in a parallelepiped of finite sizes. Clearly, this is a model that applies to large cracks that have propagated over large distance and interfaces not far from the average fracture plane.

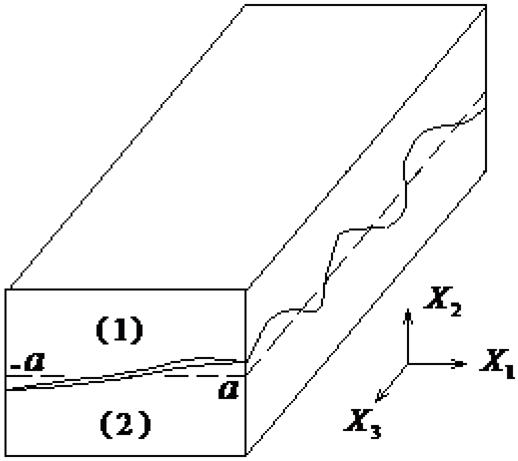

Figure 1 : *Schematic illustration of the crack front in two elastic solids (1) and (2) welded along a non-planar wavy surface that contains an interface crack. The crack fronts lie in* x_2x_3-*planes in the form* f (1)*; in this geometry, the system is subjected to mixed mode I+II+III loading with the applied tension in the* x_2-*direction. The average fracture surface (dashed) is shown perpendicular to that direction*

Our method of analysis consists of representing the crack by a continuous array of infinitesimal dislocations having the same shape as the crack front. f can be expanded in the form of a Fourier series as

$$f=\sum_n(\xi_n\sin\kappa_n x_3+\delta_n\cos\kappa_n x_3)+h\equiv\xi+h \tag{1}$$

where *n* is a positive integer; *h*, ξ_n, δ_n and κ_n are real numbers that depend on position x_1 along the crack length. From the stress fields of the dislocations, the crack-tip stresses and crack extension force (per unit length of the crack front) can be evaluated. Such analyses, with variable complexity of the crack front, exist in the case of an infinitely extended isotropic medium ([1 to 7], among others). In the case of an interface crack, mode I loading causes shear stresses corresponding to mode II and vice versa (for example, see [8, 9]). It is mandatory to have the stress fields of three types of dislocation before undertaking an analysis of the conditions of non-planar interface crack motion. In the present part I of this study, the elastic fields of sinusoidal dislocations, edges and screws with Burgers vectors $\vec{b}_I=(0,b,0)$ and $\vec{b}_{III}=(0,0,b)$ are described, with special attention to stress terms with singularities $1/x_1$ and Dirac delta function $\delta(x_1)$ that come into play in crack analyses (calculation of the crack extension force, for example). In what follows, the methodology for obtaining the dislocation elastic fields and associated calculation results are presented in Section 2 and 3, respectively. Discussion and concluding remarks form Section 4 where, in particular, the passage from the elastic fields of dislocations with a sinusoidal shape to those having the form f is indicated. The second part II of the work will deal with the elastic fields of edges (climb-type) with $\vec{b}_{II} = (b,0,0)$, crack-tip stresses and crack extension force when the non-planar interface crack is loaded in mixed mode I+II+III.

II - METHODOLOGY

We consider a dislocation lying on a non-planar interface S having the form of a corrugated sheet that separates two firmly welded elastic solids $S1$ and $S2$ of infinite sizes ***(Figure 2)***. S is defined by the point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$ and $S1$ and $S2$ occupy the regions $x_2 > \xi_n \sin \kappa_n x_3$ and $x_2 < \xi_n \sin \kappa_n x_3$, respectively. The situation is shown in ***Figure 2*** where $S1$ and $S2$ are confined for illustration purpose in a parallelepiped of finite sizes. The dislocation is located at the origin, runs indefinitely in the x_3 –direction and spreads in the $x_2 x_3$ – plane in the form

$$A_n = \xi_n \sin \kappa_n x_3 . \tag{2}$$

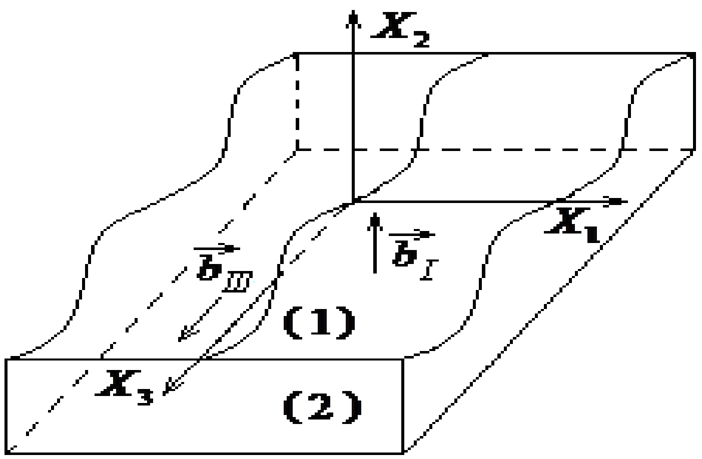

Figure 2 : *Two elastic mediums (1) and (2) welded along a non-planar sinusoidal surface and containing an interface sinusoidal dislocation at the origin. The dislocation lies in the Ox_2x_3 –plane and runs indefinitely in the x_3 –direction*

When its Burgers vector $\vec{b}_I = (0, b, 0)$ is in the x_2 –direction, the dislocation is edge on average. Because $\vec{b}_I$ is in the plane of location of the dislocation, this is a glide-type edge dislocation. With the Burgers vector $\vec{b}_{III} = (0, 0, b)$ in the x_3 –direction, the dislocation in ***Figure 2*** is screw on average. This is the aim of the present study to provide displacement $\vec{u}^{(m)}$ and stress $(\sigma)^{(m)}$ fields of these types of interface dislocation. The solution methodology is that of [10, 11]. The elastic fields $(\vec{u}^{(m)}, (\sigma)^{(m)})$ are assumed to be the difference between two quantities $(\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty})$ and $(\vec{u}^{(m)W}, (\sigma)^{(m)W})$:

$$\begin{aligned} &\vec{u}^{(m)} = \vec{u}^{(m)\infty} - \vec{u}^{(m)W} \\ &(\sigma)^{(m)} = (\sigma)^{(m)\infty} - (\sigma)^{(m)W} \end{aligned} . \qquad (3)$$

The former with ∞ corresponds to the fields of a sinusoidal dislocation (edge or screw) in an infinitely extended homogeneous solid (m); the latter with W satisfies the equations of equilibrium and is constructed in such a way that :
(a) $(\vec{u}^{(m)}, (\sigma)^{(m)})$ are continuous at the crossing of the interface, implying that

$$\begin{aligned} &\Delta\vec{u}^{\infty}(P_S) \equiv \vec{u}^{(2)\infty} - \vec{u}^{(1)\infty} = \vec{u}^{(2)W} - \vec{u}^{(1)W} \equiv \Delta\vec{u}^{W}(P_S) \\ &(\Delta\sigma)^{\infty}(P_S) \equiv (\sigma)^{(2)\infty} - (\sigma)^{(1)\infty} = (\sigma)^{(2)W} - (\sigma)^{(1)W} \equiv (\Delta\sigma)^{W}(P_S); \end{aligned} \qquad (4)$$

(b) $(\vec{u}^{(m)}, (\sigma)^{(m)})$ tends to $(\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty})$ when one moves far away from the interface in the x_2 –direction. This means that

$$\begin{aligned} &\vec{u}^{(m)W} \to 0 \\ &(\sigma)^{(m)W} \to 0 \end{aligned} \qquad (5)$$

when $|x_2| \to \infty$. $(\vec{u}^{(m)\infty}, (\sigma)^{(m)\infty})$ may be taken from [5, 6]; they are given to linear expressions with respect to ξ_n. The associated terms $(\vec{u}^{(0)(m)\infty}, (\sigma)^{(0)(m)\infty})$ of zero order correspond to the fields of a straight dislocation and second terms $(\vec{u}^{A_n(m)\infty}, (\sigma)^{A_n(m)\infty})$ are proportional to either A_n or its spatial derivative $\partial A_n / x_3$. Hence we have

$$\begin{aligned} &\Delta\vec{u}^{\infty}(P_S) = \Delta\vec{u}^{(0)\infty} + \Delta\vec{u}^{A_n\infty} \\ &(\Delta\sigma)^{\infty}(P_S) = (\Delta\sigma)^{(0)\infty} + (\Delta\sigma)^{A_n\infty} \end{aligned} , \qquad (6)$$

on the interface point $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$; Appendix A below gives the complete list, component by component, for the sinusoidal screw dislocation; the corresponding values for the glide-type sinusoidal edge dislocation have been given in [10, 11]. ($\vec{u}^{(m)W}, (\sigma)^{(m)W}$) are obtained with the help of Galerkin vectors; these are available for the edge dislocation (see [10, 11]). For the screw dislocation, we arrive at Galerkin vectors $\vec{V}$ with only one non-zero x_3 – component, arranged in the form

$$V_3(\vec{x}) = \overline{\alpha}_3(\vec{k})e^{i\vec{k}.\vec{x}} + \overline{\beta}_3(\vec{k})x_2 e^{i\vec{k}.\vec{x}} \tag{7}$$

under the condition $\vec{k}^2 = k_1^2 + k_2^2 + k_3^2 = 0$ that ensures the biharmonicity of V_3. For V_3 to cancel far from the interface, we write

$$k_2 = k_2^{(m)} \equiv (-1)^{m-1} i\sqrt{k_1^2 + k_3^2} \tag{8}$$

with $m = 1$ when $x_2 > \xi_n \sin\kappa_n x_3$ (half-space 1) and $m = 2$ when $x_2 < \xi_n \sin\kappa_n x_3$ (half-space 2). We use the notations

$$\vec{k}^{(m)} \equiv (k_1, k_2^{(m)}, k_3),\ \overline{\alpha}_3^{(m)} \equiv \overline{\alpha}_3(\vec{k}^{(m)}),\ \overline{\beta}_3^{(m)} \equiv \overline{\beta}_3(\vec{k}^{(m)});$$

hence for half-space 1 ($x_2 > \xi_n \sin\kappa_n x_3$), solid (1)

$$V_3(\vec{x}) \equiv V_3^{(1)}(\vec{x}) = \overline{\alpha}_3^{(1)} e^{i\vec{k}^{(1)}.\vec{x}} + \overline{\beta}_3^{(1)} x_2 e^{i\vec{k}^{(1)}.\vec{x}}$$

and for half-space 2 ($x_2 < \xi_n \sin\kappa_n x_3$), solid (2)

$$V_3(\vec{x}) \equiv V_3^{(2)}(\vec{x}) = \overline{\alpha}_3^{(2)} e^{i\vec{k}^{(2)}.\vec{x}} + \overline{\beta}_3^{(2)} x_2 e^{i\vec{k}^{(2)}.\vec{x}}.$$

The elastic fields corresponding to V_3 (7) may be first calculated; then, more general forms $\vec{u}^{(m)V}$ and $(\sigma)^{(m)V}$ are constructed from the previous ones by superposition over k_1 and k_3; we may write

$$\begin{aligned} \vec{u}^{(m)V} &= \vec{u}^{(m)A} + \vec{u}^{(m)B} = \vec{u}^{(m)A+B} \\ (\sigma)^{(m)V} &= (\sigma)^{(m)A} + (\sigma)^{(m)B} = (\sigma)^{(m)A+B} \end{aligned} \tag{9}$$

where terms with A and B refer to $\overline{\alpha}_3$ and $\overline{\beta}_3$ in (7) respectively. For $\vec{u}^{(m)V}$ and $(\sigma)^{(m)V}$ to conform with $\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$, the summation over k_1 is continuous and that over k_3 is discrete. k_3 takes three values : $-\kappa_n$, 0, κ_n. The fields corresponding to $k_3 = 0$ are denoted $\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$ and terms associated with $k_3 = -\kappa_n$ and κ_n are merged to form expressions denoted by $\vec{u}^{A_n(m)V}$ and $(\sigma)^{A_n(m)V}$; this is made possible by requiring that

$$\bar{\alpha}_3^{(m)}(\kappa_n) \equiv -\bar{\alpha}_3^{(m)}(-\kappa_n),\ \bar{\beta}_3^{(m)}(\kappa_n) \equiv -\bar{\beta}_3^{(m)}(-\kappa_n). \tag{10}$$

In (20), $\bar{\alpha}_3^{(m)}(\kappa_n)$ stands for $\bar{\alpha}_3(k_1, k_2^{(m)}, \kappa_n)$. We write

$$\begin{aligned} \vec{u}^{(m)V} &= \vec{u}^{(0)(m)V} + \vec{u}^{A_n(m)V} \\ (\sigma)^{(m)V} &= (\sigma)^{(0)(m)V} + (\sigma)^{A_n(m)V} \end{aligned} \tag{11}$$

introducing subsequently the notation $\vec{u}^{(0)(m)A}$, $\vec{u}^{A_n(m)A}$, $\vec{u}^{(0)(m)B}$, $\vec{u}^{A_n(m)B}$ and even for the stress. $\vec{u}^{(0)(m)V}$ and $(\sigma)^{(0)(m)V}$ are x_3 – independent; $\vec{u}^{A_n(m)V}$ and $(\sigma)^{A_n(m)V}$ are proportional to the sinusoid $A_n(x_3)$ or to its spatial derivative $\partial A_n / \partial x_3$. Here also, for points P_S on the interface, $\Delta\vec{u}^V$ and $(\Delta\sigma)^V$ are expanded up to terms of first order with respect to $x_2 = \xi$ in a similar manner as in (A.2) (see Appendix A) for $\Delta\vec{u}^\infty$ and $(\Delta\sigma)^\infty$. Requiring $\Delta\vec{u}^V = \Delta\vec{u}^\infty$ and $(\Delta\sigma)^V = (\Delta\sigma)^\infty$ lead to the following equations, writing first the conditions corresponding to $k_3 = 0$ (i.e. $\Delta u_i^{(0)V} = \Delta u_i^{(0)\infty}$ and $\Delta\sigma_{ij}^{(0)V} = \Delta\sigma_{ij}^{(0)\infty}$).

$$\Delta u_3^{(0)V} = \Delta u_3^{(0)\infty} \Rightarrow$$

$$\frac{\bar{\beta}_3^{(2)}}{C_2} + \frac{\bar{\beta}_3^{(1)}}{C_1} = 0 \qquad (a)$$

$$\Delta\sigma_{13}^{(0)V} = \Delta\sigma_{13}^{(0)\infty} \text{ and } \Delta\sigma_{23}^{(0)V} = \Delta\sigma_{23}^{(0)\infty} \Rightarrow$$

$$(1-\nu_2)\bar{\beta}_3^{(2)} + (1-\nu_1)\bar{\beta}_3^{(1)} = 0 \qquad (b)$$

$$(1-\nu_2)\bar{\beta}_3^{(2)} - (1-\nu_1)\bar{\beta}_3^{(1)} = (Q_c - Q_b)\frac{\mathrm{sgn}(k_1)}{k_1^2} \qquad (c) \qquad (12)$$

where $C_m = b\mu_m / 2\pi(1-\nu_m)$, $Q_b = i(C_2 - C_1)/4$, $Q_c = i(\nu_2 C_2 - \nu_1 C_1)/4$; $\mathrm{sgn}(k_1) = k_1/|k_1|$; μ_m and ν_m are shear modulus and Poisson's ratio. In equations (12 *a* to *c*) above, $\bar{\beta}_3^{(m)}$ stands for $\bar{\beta}_3(k_1, k_2^{(m)}, k_3 = 0)$. Other elastic field components are zero. The conditions corresponding to $\Delta u_i^{A_n V} = \Delta u_i^{A_n\infty}$ and $\Delta\sigma_{ij}^{A_n V} = \Delta\sigma_{ij}^{A_n\infty}$ are now listed as :

$$\Delta u_1^{A_n V} = \Delta u_1^{A_n \infty} \Rightarrow$$

$$\frac{\overline{\alpha}_3^{(2)}}{\mu_2} - \frac{\overline{\alpha}_3^{(1)}}{\mu_1} = \frac{bC_\nu \xi_n}{8\pi} \frac{k_1}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} \quad (a)$$

$$\sqrt{k_1^2 + \kappa_n^2}\left(\frac{\overline{\alpha}_3^{(2)}}{\mu_2} + \frac{\overline{\alpha}_3^{(1)}}{\mu_1}\right) + \frac{\overline{\beta}_3^{(2)}}{\mu_2} - \frac{\overline{\beta}_3^{(1)}}{\mu_1} = 0 \quad (b)$$

$$\Delta u_2^{A_n V} = \Delta u_2^{A_n \infty} \Rightarrow (b) \text{ above and}$$

$$\sqrt{k_1^2 + \kappa_n^2}\left(\frac{\overline{\alpha}_3^{(2)}}{\mu_2} - \frac{\overline{\alpha}_3^{(1)}}{\mu_1}\right) + 2\left(\frac{\overline{\beta}_3^{(2)}}{\mu_2} + \frac{\overline{\beta}_3^{(1)}}{\mu_1}\right) = -\frac{bC_\nu \xi_n}{8\pi} \frac{k_1}{k_1^2 + \kappa_n^2} \quad (c)$$

$$\Delta u_3^{A_n V} = \Delta u_3^{A_n \infty} \Rightarrow$$

$$\kappa_n^2\left(\frac{\overline{\alpha}_3^{(2)}}{\mu_2} - \frac{\overline{\alpha}_3^{(1)}}{\mu_1}\right) + 4\sqrt{k_1^2 + \kappa_n^2}\left(\frac{(1-\nu_2)\overline{\beta}_3^{(2)}}{\mu_2} + \frac{(1-\nu_1)\overline{\beta}_3^{(1)}}{\mu_1}\right) = \frac{bC_\nu \xi_n \kappa_n^2}{8\pi} \frac{k_1}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} \quad (d)$$

$$\kappa_n^2\sqrt{k_1^2 + \kappa_n^2}\left(\frac{\overline{\alpha}_3^{(2)}}{\mu_2} + \frac{\overline{\alpha}_3^{(1)}}{\mu_1}\right) + \frac{\overline{\beta}_3^{(2)}}{\mu_2}\left[\kappa_n^2 + 4(1-\nu_2)(k_1^2 + \kappa_n^2)\right] - \frac{\overline{\beta}_3^{(1)}}{\mu_1}\left[\kappa_n^2 + 4(1-\nu_1)(k_1^2 + \kappa_n^2)\right] = 0 \quad (e)$$

$$\Delta \sigma_{11}^{A_n V} = \Delta \sigma_{11}^{A_n \infty} \Rightarrow$$

$$k_1^2\left(\overline{\alpha}_3^{(2)} - \overline{\alpha}_3^{(1)}\right) + 2\sqrt{k_1^2 + \kappa_n^2}\left(\nu_2\overline{\beta}_3^{(2)} + \nu_1\overline{\beta}_3^{(1)}\right) = -\frac{iQ_b \xi_n k_1 (k_1^2 + 2\kappa_n^2)}{\left(k_1^2 + \kappa_n^2\right)^{3/2}} \quad (f)$$

$$k_1^2\sqrt{k_1^2 + \kappa_n^2}\left(\overline{\alpha}_3^{(2)} + \overline{\alpha}_3^{(1)}\right) + [(1+2\nu_2)k_1^2 + 2\nu_2\kappa_n^2]\overline{\beta}_3^{(2)} - [(1+2\nu_1)k_1^2 + 2\nu_1\kappa_n^2]\overline{\beta}_3^{(1)} = 0 \quad (g)$$

$$\Delta \sigma_{22}^{A_n V} = \Delta \sigma_{22}^{A_n \infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2}\left(\overline{\alpha}_3^{(2)} - \overline{\alpha}_3^{(1)}\right) + 2(1-\nu_2)\overline{\beta}_3^{(2)} + 2(2-\nu_1)\overline{\beta}_3^{(1)} = \frac{i(2Q_c - Q_b)\xi_n k_1}{k_1^2 + \kappa_n^2} \quad (h)$$

$$\sqrt{k_1^2 + \kappa_n^2}\left(\overline{\alpha}_3^{(2)} + \overline{\alpha}_3^{(1)}\right) + (3-2\nu_2)\overline{\beta}_3^{(2)} - (3-2\nu_1)\overline{\beta}_3^{(1)} = 0 \quad (i)$$

$$\Delta \sigma_{33}^{A_n V} = \Delta \sigma_{33}^{A_n \infty} \Rightarrow$$

$$\kappa_n^2\left(\overline{\alpha}_3^{(2)} - \overline{\alpha}_3^{(1)}\right) + 2\sqrt{k_1^2 + \kappa_n^2}\left((2-\nu_2)\overline{\beta}_3^{(2)} + (2-\nu_1)\overline{\beta}_3^{(1)}\right)$$

$$= -\frac{iQ_b\xi_n k_1(2k_1^2+\kappa_n^2)}{\left(k_1^2+\kappa_n^2\right)^{3/2}} \qquad (j)$$

$$\kappa_n^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_3^{(2)}+\overline{\alpha}_3^{(1)}\right)+[(5-2\nu_2)\kappa_n^2+2(2-\nu_2)k_1^2]\overline{\beta}_3^{(2)}$$
$$-[(5-2\nu_1)\kappa_n^2+2(2-\nu_1)k_1^2]\overline{\beta}_3^{(1)}=0 \qquad (k)$$

$$\Delta\sigma_{12}^{A_nV}=\Delta\sigma_{12}^{A_n\infty}\Rightarrow$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_3^{(2)}-\overline{\alpha}_3^{(1)}\right)+2(\overline{\beta}_3^{(2)}+\overline{\beta}_3^{(1)})=\frac{i\xi_n[(Q_c-2Q_b)k_1^2+(Q_c-Q_b)\kappa_n^2]}{k_1(k_1^2+\kappa_n^2)} \qquad (l)$$

$$\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_3^{(2)}+\overline{\alpha}_3^{(1)}\right)+\overline{\beta}_3^{(2)}-\overline{\beta}_3^{(1)}=0 \qquad (m)$$

$$\Delta\sigma_{13}^{A_nV}=\Delta\sigma_{13}^{A_n\infty}\Rightarrow$$

$$\kappa_n^2\left(\overline{\alpha}_3^{(2)}-\overline{\alpha}_3^{(1)}\right)+2\sqrt{k_1^2+\kappa_n^2}\left((1-\nu_2)\overline{\beta}_3^{(2)}+(1-\nu_1)\overline{\beta}_3^{(1)}\right)$$
$$=\frac{i\xi_n\left[Q_b\kappa_n^2k_1^2+(Q_c-Q_b)(k_1^2+\kappa_n^2)^2\right]}{k_1\left(k_1^2+\kappa_n^2\right)^{3/2}} \qquad (n)$$

$$\kappa_n^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_3^{(2)}+\overline{\alpha}_3^{(1)}\right)+[(3-2\nu_2)\kappa_n^2+2(1-\nu_2)k_1^2]\overline{\beta}_3^{(2)}$$
$$-[(3-2\nu_1)\kappa_n^2+2(1-\nu_1)k_1^2]\overline{\beta}_3^{(1)}=0 \qquad (o)$$

$\Delta\sigma_{23}^{A_nV}=\Delta\sigma_{23}^{A_n\infty}\Rightarrow(o)$ above and

$$\kappa_n^2\sqrt{k_1^2+\kappa_n^2}\left(\overline{\alpha}_3^{(2)}-\overline{\alpha}_3^{(1)}\right)+2[(2-\nu_2)\kappa_n^2+(1-\nu_2)k_1^2]\overline{\beta}_3^{(2)}$$
$$+2[(2-\nu_1)\kappa_n^2+(1-\nu_1)k_1^2]\overline{\beta}_3^{(1)}=\frac{i\xi_nk_1[(Q_c-Q_b)k_1^2+(Q_c-2Q_b)\kappa_n^2]}{k_1^2+\kappa_n^2} \qquad (p) \quad (13)$$

Next, we are concerned with satisfying boundary conditions : (12) leads to the displacement and stress fields due to an interface straight screw dislocation ($\vec{b}_{III}=(0,0,b)$) parallel to the x_3-direction at the origin; the interface is the Ox_1x_3-plane. (13) provides the complementary terms (to first order in ξ_n) in the elastic fields of an interfacial sinusoidal screw dislocation.

III - CALCULATION RESULTS

III-1. Displacement and stress fields of an interface glide-type sinusoidal edge dislocation

When ξ_n is small, the elastic fields (displacement $\vec{u}^{(m)}$ and stress $(\sigma)^{(m)}$) at an arbitrary position $\vec{x} = (x_1, x_2, x_3)$ may be expressed (to linear terms in ξ_n) as

$$\begin{aligned}\vec{u}^{(m)} &= \vec{u}^{(0)(m)} + \vec{u}^{A_n(m)} \\ (\sigma)^{(m)} &= (\sigma)^{(0)(m)} + (\sigma)^{A_n(m)},\end{aligned} \tag{14}$$

with m=1 and 2 for medium S1 and S2, respectively. $\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$ are of zero order, independent of x_3, and correspond to the elastic fields of a straight edge dislocation lying on the planar Ox_1x_3 – interface. These are given under continuity requirement of the fields on crossing the interface [10, 11]. $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ are oscillating fields proportional to either A_n or its spatial derivative $\partial A_n / \partial x_3$ with respect to x_3, written [11] in the forms

$$\begin{aligned}\vec{u}^{A_n(m)} &= \vec{u}^{A_n(m)\infty} - \vec{u}^{A_n(m)W} \\ (\sigma)^{A_n(m)} &= (\sigma)^{A_n(m)\infty} - (\sigma)^{A_n(m)W}.\end{aligned} \tag{15}$$

Here, expressions with ∞ are associated with a sinusoidal edge dislocation in an infinitely extended homogeneous medium (m) with the equal elastic constants (see oscillating fields given in [5, 6, 12]); second terms with W read

$$\begin{aligned}\vec{u}^{A_n(m)W} &= \eta_a^{A_n(m)}\vec{u}_a^{A_n(m)V} + \eta_b^{A_n(m)}\vec{u}_b^{A_n(m)V} + \eta_c^{A_n(m)}\vec{u}_c^{A_n(m)V} + \eta_d^{A_n(m)}\vec{u}_d^{A_n(m)V} \\ (\sigma)^{A_n(m)W} &= \eta_a^{A_n(m)}(\sigma)_a^{A_n(m)V} + \eta_b^{A_n(m)}(\sigma)_b^{A_n(m)V} + \eta_c^{A_n(m)}(\sigma)_c^{A_n(m)V} + \eta_d^{A_n(m)}(\sigma)_d^{A_n(m)V}\end{aligned} \tag{16}$$

$\vec{u}_{a\text{ to }d}^{A_n(m)V}$ and $(\sigma)_{a\text{ to }d}^{A_n(m)V}$ are given in [11]; $\eta_{a\text{ to }d}^{A_n(m)}$ are real determined essentially by continuity conditions of the fields $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ on the interface. These lead to a number of equations denoted by $e_i^{A_n}$ (see relation (43) in [11]). We follow the treatment in [11] (same definitions and notations) but introduce some changes that follow; in this section, we use the notation *Eqn* (N), N integer, to designate an equation (N) in [11] :

(i) Retaining only terms proportional to $1/x_1$, $u_2^{A_n(m)}$ is written as

$$u_2^{A_n(m)}(x_1, 0, x_3) = -\frac{A_n}{2} e_3^{A_n *} \frac{1}{x_1} \tag{17}$$

were

$$e_3^{A_n *} \equiv \frac{1}{\mu_m}\Big\{C_m(1-2\nu_m) + \eta_a^{A_n(m)} 2a_{1a}^{(m)} + \eta_b^{A_n(m)}(-1)^m(1-2\nu_m)2iQ_b \\ + \eta_c^{A_n(m)} 2i\left[\sum_{i=1}^{3} a_{ic}^{(m)} + (-1)^{m-1}2\rho_m(1-2\nu_m)\big(b_{1c} - b_{2c} + b_{3c}\big)\right]$$

$$+\eta_d^{A_n(m)}2i\left[a_{1d}^{(m)}+a_{2d}^{(m)}+(-1)^{m-1}4(1-2\nu_m)\bar{b}^{(m)}\right]\Big\}.$$

$e_3^{A_n*}$ is constant with m=1 and 2. $e_3^{A_n*}$ is used in place of $e_3^{A_n}$ in *Eqn* (43). We stress that $e_3^{A_n}$ is proportional to the modified Bessel function $K_1[\kappa_n|x_1|]$ only that has a singularity of the type $1/x_1$ but additional terms with $1/x_1$ do exist in $u_2^{A_n(m)}$. Taking into account all the singularity terms with $1/x_1$ leads to $e_3^{A_n*}$ above. (*ii*) $e_{72}^{A_n}$ in *Eqn* (43) is proportional to the modified Bessel function $K_0[\kappa_n|x_1|]$ only that has a singularity of the type $\ln|x_1|$; however, there are other terms with $\ln|x_1|$ in $\sigma_{12}^{A_n(m)}$. Collecting all the terms with $\ln|x_1|$ and setting the coefficient of $\ln|x_1|$ constant with *m* corresponds to $e_{72}^{A_n*}$ constant with *m* with

$$\begin{aligned} e_{72}^{A_n*} \equiv -\nu_m C_m + \eta_a^{A_n(m)}2a_{1a}^{(m)} + \eta_b^{A_n(m)}(-1)^{m-1}iQ_b\left(2+\nu_m-\rho_m\right) \\ +\eta_c^{A_n(m)}2i\left[a_{2c}^{(m)}+\theta a_{3c}^{(m)}+(-1)^{m-1}2\nu_m\rho_m(b_{2c}-\theta b_{3c})\right] \\ +\eta_d^{A_n(m)}2i\left[a_{2d}^{(m)}+(-1)^m 2\nu_m\bar{b}^{(m)}\right]. \end{aligned} \qquad (18)$$

$e_{72}^{A_n*}$ is at present considered in place of $e_{72}^{A_n}$. (*iii*) We add a new equation $e_9^{A_n*}$ corresponding to the condition that $\sigma_{23}^{A_n(m)}(x_1,0,x_3)$ is constant with m=1 and 2. Restricting ourselves to all the terms with $1/x_1$ only, we obtain

$$\sigma_{23}^{A_n(m)}(x_1,0,x_3) \equiv \frac{\partial A_n}{\partial x_3}\frac{e_9^{A_n*}}{x_1} \qquad (19)$$

with

$$\begin{aligned} e_9^{A_n*} \equiv \nu_m C_m - \eta_a^{A_n(m)}2a_{1a}^{(m)} + \eta_b^{A_n(m)}(-1)^m 2iQ_b \\ -\eta_c^{A_n(m)}2i\left[\sum_{i=1}^{3}a_{ic}^{(m)}+(-1)^m 2\rho_m\nu_m\left(b_{1c}-b_{2c}+b_{3c}\right)\right] \\ -\eta_d^{A_n(m)}2i\left[a_{1d}^{(m)}+a_{2d}^{(m)}+(-1)^m 4\nu_m\bar{b}^{(m)}\right]. \end{aligned}$$

We stress that all the others $e_i^{A_n}$ in *Eqn* (43) are unchanged. (*iv*) In the resolution procedure [11], it is required to choose four equations (*Ei*), *i*=1 to 4, corresponding to four independent equations $e_j^{A_n}(1)=e_j^{A_n}(2)$. A choice corresponds to *Eqn* (51); (*E*1) and (*E*2), there, are at present replaced by (*E*1)* and (*E*2)* below.

(*E*1)*: $e_9^{A_n*}(2)+e_{72}^{A_n*}(2)=e_9^{A_n*}(1)+e_{72}^{A_n*}(1)$

$(E2)^*$: $e_9^{A_n*}(2) = e_9^{A_n*}(1)$ (20)

(E3) and (E4) are unchanged. The identifications corresponding to (E1)* and (E2)*, to be used in place of (E1) and (E2) in *Eqn* (52), are :

$$
\begin{aligned}
b_1 &= 2i\left(a_{1d}^{(2)} + 2\nu_2\bar{b}^{(2)}\right)D,\\
a_{11} &= -2i\left(a_{1d}^{(2)} + 2\nu_2\bar{b}^{(2)}\right)D_a,\\
a_{12} &= -iQ_b(\nu_2-\nu_1)B_b - 2i\left(a_{1c}^{(2)} + (1-\theta)a_{3c}^{(2)} + 2\nu_1\nu_2\left[b_{1c} - 2b_{2c} + (1+\theta)b_{3c}\right]\right)C_b\\
&\qquad - 2i\left(a_{1d}^{(2)} + 2\nu_2\bar{b}^{(2)}\right)D_d + iQ_b(\nu_2-\nu_1),\\
a_{13} &= -iQ_b(\nu_2-\nu_1)B_c - 2i\left(a_{1c}^{(2)} + (1-\theta)a_{3c}^{(2)} + 2\nu_1\nu_2\left[b_{1c} - 2b_{2c} + (1+\theta)b_{3c}\right]\right)C_c\\
&\quad - 2i\left(a_{1d}^{(2)} + 2\nu_2\bar{b}^{(2)}\right)D_c - 2i\left(a_{1c}^{(1)} + (1-\theta)a_{3c}^{(1)} - 2\nu_1\nu_2\left[b_{1c} - 2b_{2c} + (1+\theta)b_{3c}\right]\right),\\
a_{14} &= -2i\left(a_{1d}^{(2)} + 2\nu_2\bar{b}^{(2)}\right)D_d - 2i\left(a_{1d}^{(1)} - 2\nu_1\bar{b}^{(1)}\right);
\end{aligned}
$$

$$
\begin{aligned}
b_2 &= \nu_1C_1 - \nu_2C_2 + 2a_{1a}^{(2)}A + 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}^{(2)}\right)D,\\
a_{21} &= -2a_{1a}^{(2)}A_a - 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}^{(2)}\right)D_a - 2a_{1a}^{(1)},\\
a_{22} &= -2a_{1a}^{(2)}A_b + 2iQ_bB_b - 2i\sum_{s=1}^{3}\left[a_{sc}^{(2)} + 2\nu_1\nu_2(-1)^{s-1}b_{sc}\right]C_b\\
&\qquad - 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}^{(2)}\right)D_b - 2iQ_b,\\
a_{23} &= -2a_{1a}^{(2)}A_c + 2iQ_bB_c - 2i\sum_{s=1}^{3}\left[a_{sc}^{(2)} + 2\nu_1\nu_2(-1)^{s-1}b_{sc}\right]C_c\\
&\qquad - 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}^{(2)}\right)D_c + 2i\sum_{s=1}^{3}\left[a_{sc}^{(1)} + 2\nu_1\nu_2(-1)^{s}b_{sc}\right],\\
a_{24} &= -2a_{1a}^{(2)}A_d - 2i\left(a_{1d}^{(2)} + a_{2d}^{(2)} + 4\nu_2\bar{b}^{(2)}\right)D_d + 2i\left(a_{1d}^{(1)} + a_{2d}^{(1)} - 4\nu_1\bar{b}^{(1)}\right).
\end{aligned} \tag{21}
$$

With changes (i) to (i4) above, the resolution methodology [11] can be followed, step by step, with no more modifications in the various listed relations there (i.e. *Eqn* (43) to *Eqn* (56) included, and in addition, all the expressions in the associated Appendix). Again, we stress that the singularity term in $\sigma_{23}^{A_n(m)}(x_1, 0, x_3)$ is given by (19) above with $e_9^{A_n*}$ and not *Eqn* (57) with $e_{72}^{A_n}$. The solutions $\eta_b^{A_n(m)}$ and $\eta_c^{A_n(m)}$ (m=1 and 2) have been listed in *Eqn* (53) to *Eqn* (55). Using the two first equations in *Eqn* (47), we can write

$$
\eta_a^{A_n(1)} = \frac{a_{33}(a_{24}b_1 - a_{14}b_2) + \left[a_{33}(a_{14}a_{22} - a_{24}a_{12}) - a_{32}(a_{14}a_{23} - a_{24}a_{13})\right]\eta_b^{A_n(1)}}{a_{33}(a_{24}a_{11} - a_{14}a_{21})},
$$

$$\eta_d^{A_n(1)} = \frac{a_{33}(a_{21}b_1 - a_{11}b_2) + \left[a_{33}(a_{11}a_{22} - a_{21}a_{12}) - a_{32}(a_{11}a_{23} - a_{21}a_{13})\right]\eta_b^{A_n(1)}}{a_{33}(a_{14}a_{21} - a_{24}a_{11})}. \tag{22}$$

$\eta_a^{A_n(2)}$ and $\eta_d^{A_n(2)}$ are obtained using *Eqn* (44):

$$\eta_a^{A_n(2)} = A + A_a\eta_a^{A_n(1)} + A_b\eta_b^{A_n(1)} + A_c\eta_c^{A_n(1)} + A_d\eta_d^{A_n(1)},$$

$$\eta_d^{A_n(2)} = D + D_a\eta_a^{A_n(1)} + D_b\eta_b^{A_n(1)} + D_c\eta_c^{A_n(1)} + D_d\eta_d^{A_n(1)}. \tag{23}$$

In summary, all the $\eta_{a\text{ to }d}^{A_n(m)}$ (m=1 and 2) have been given, permitting one to express the oscillating parts (15) of the elastic fields $\vec{u}^{(m)}$ and $(\sigma)^{(m)}$ (14). We stress that in our previous work [11], we have imposed coefficients of modified Bessel functions $K_0[\kappa_n|x_1|]$ and $K_1[\kappa_n|x_1|]$ constant with m in the elastic fields $u_2^{A_n(m)}(x_1,0,x_3)$, $\sigma_{12}^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{23}^{A_n(m)}(x_1,0,x_3)$ whilst the present study focuses on functions $\ln|x_1|$ and $1/x_1$, respectively. Both procedures are self-consistent and should yield the equal $\eta_{a\text{ to }d}^{A_n(m)}$.

III-2. Displacement and stress fields of an interface sinusoidal screw dislocation

III-2-1. Displacement and stress fields due to an interface straight screw dislocation

Two distinct values for $\overline{\beta}_3^{(m)}$ are extracted from (12); these are :

$$(a)\ \overline{\beta}_3^{(m)} = \left(\delta_{1m} - \frac{C_2}{C_1}\delta_{2m}\right)\frac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \overline{\beta}_{3a}^{(m)}$$

$$(b)\ \overline{\beta}_3^{(m)} = (-1)^{m-1}\frac{(Q_b - Q_c)}{2(1-\nu_m)}\frac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \overline{\beta}_{3b}^{(m)} \tag{24}$$

where δ_{ij} is the Kronecker delta. $\overline{\beta}_{3a}^{(m)}$ is obtained using (12 *a*) and continuity requirement for $\vec{u}^{(0)(m)}$ at the crossing of the interface at $x_2 = 0$. $\overline{\beta}_{3b}^{(m)}$ is obtained from (12 *b* and *c*) associated with the stresses. None of these values satisfies the entire (12). The associated elastic fields denoted $\vec{u}_{a\text{ and }b}^{(0)(m)V}$ and $(\sigma)_{a\text{ and }b}^{(0)(m)V}$ are displayed below. A superposition of these partial fields will provide the complete form of the solution. We have at position $\vec{x} = (x_1, x_2, x_3)$:

$$\begin{aligned}
&\delta_{ja}u_{3a}^{(0)(m)V}+\delta_{jb}u_{3b}^{(0)(m)V}=\frac{1}{\mu_m}\left(\delta_{ja}(-1)^m V_a^{(m)}+\delta_{jb}2i(Q_c-Q_b)\right)\tan^{-1}\frac{x_1}{|x_2|},\\
&\delta_{ja}\sigma_{23a}^{(0)(m)V}+\delta_{jb}\sigma_{23b}^{(0)(m)V}=\left(\delta_{ja}V_a^{(m)}+\delta_{jb}(-1)^m 2i(Q_c-Q_b)\right)\frac{x_1}{r^2},\\
&\delta_{ja}\sigma_{13a}^{(0)(m)V}+\delta_{jb}\sigma_{13b}^{(0)(m)V}=\left(\delta_{ja}(-1)^m V_a^{(m)}+\delta_{jb}2i(Q_c-Q_b)\right)\left(\frac{|x_2|}{r^2}+\pi\delta_A(x_2)\delta(x_1)\right)
\end{aligned}\qquad(25)$$

where $V_a^{(m)}=4i(1-\nu_m)(\delta_{1m}-\delta_{2m}C_2/C_1)$, $j = a$ and b and $\delta(x_1)$ is the Dirac delta function; here δ_A has the following definition : $\delta_A(x_2)=0$ when $x_2\neq 0$ and $\delta_A(x_2)=1$ when $x_2=0$; $r^2=x_1^2+x_2^2$. The other elastic fields are zero. We define the elastic fields $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ of an interface straight screw dislocation as

$$\begin{aligned}
&\vec{u}^{(0)(m)}=\vec{u}^{(0)(m)\infty}-\vec{u}^{(0)(m)W}\\
&(\sigma)^{(0)(m)}=(\sigma)^{(0)(m)\infty}-(\sigma)^{(0)(m)W}
\end{aligned}\qquad(26)$$

with

$$\begin{aligned}
&\vec{u}^{(0)(m)W}=\eta_a^{(m)}\vec{u}_a^{(0)(m)V}+\eta_b^{(m)}\vec{u}_b^{(0)(m)V}\\
&(\sigma)^{(0)(m)W}=\eta_a^{(m)}(\sigma)_a^{(0)(m)V}+\eta_b^{(m)}(\sigma)_b^{(0)(m)V}\ .
\end{aligned}\qquad(27)$$

Again $\vec{u}^{(0)(m)\infty}$ and $(\sigma)^{(0)(m)\infty}$ are due to a straight screw $\vec{b}_{III}=(0,0,b)$ parallel to the x_3-direction at the origin in an infinite medium (see [5, 6] for example); $\vec{u}_{a\text{ and }b}^{(0)(m)V}$ and $(\sigma)_{a\text{ and }b}^{(0)(m)V}$ are given in (25). $\eta_{a\text{ and }b}^{(m)}$ are real, to be determined by the condition that the elastic fields satisfy the following relations :

- $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ are continuous when crossing the Ox_1x_3- plane.
- $\oint_\Gamma du_3^{(0)(m)}=b$ for a closed contour Γ in x_1x_2 encircling the dislocation.
- $\vec{u}^{(0)(m)W}$ vanish far from the interface (i.e. when $|x_2|\to\infty$).

The last condition is fulfilled because all the $\vec{u}_{a\text{ and }b}^{(0)(m)V}$ (25) vanish when $|x_2|\to\infty$. Also $(\sigma)^{(0)(m)\infty}$ and $(\sigma)_{a\text{ and }b}^{(0)(m)V}$ vanish at infinity. Under such conditions, $\vec{u}^{(0)(m)}(\vec{x})$ and $(\sigma)^{(0)(m)}(\vec{x})$ do correspond to an interface straight screw dislocation. Next, we express the quantities involved in the above mentioned requirements and proceed to satisfy these.

$$u_3^{(0)(1)} = u_3^{(0)(2)} \text{ and } \oint_\Gamma du_3^{(0)(m)} = b \Rightarrow$$
$$\left(\eta_a^{(m)}(-1)^m V_a^{(m)} + \eta_b^{(m)} 2i(Q_c - Q_b)\right)/\mu_m \equiv e_1^{'}$$
$$\sigma_{13}^{(0)(1)} = \sigma_{13}^{(0)(2)} \Rightarrow$$
$$\eta_a^{(m)}(-1)^m V_a^{(m)} + \eta_b^{(m)} 2i(Q_c - Q_b) \equiv e_2^{'}$$
$$\sigma_{23}^{(0)(1)} = \sigma_{23}^{(0)(2)} \Rightarrow$$
$$D_m - \eta_a^{(m)} V_a^{(m)} + \eta_b^{(m)}(-1)^{m-1} 2i(Q_c - Q_b) \equiv e_3^{'} \qquad (28)$$

where $D_m = b\mu_m / 2\pi = C_m(1-\nu_m)$; all $e_i^{'}$ are constant with m=1 and 2. Only expressions interconnecting the $\eta_{a \text{ and } b}^{(m)}$ may be derived from (28). But only one expression leaves unchanged elastic fields $\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$ by inverting the elastic constants. This is written as

$$\eta_a^{(m)}(-1)^m V_a^{(m)} + \eta_b^{(m)} 2i(Q_c - Q_b) \equiv \frac{(\mu_1\delta_{1m} + \mu_2\delta_{2m})(D_2 - D_1)}{\mu_1 + \mu_2}, \; m\text{=1 and 2.} \qquad (29)$$

It may be obtained using $e_{1 \text{ and } 3}^{'}$(28) in the form

$$D_m + (-1)^{m-1}\mu_m e_1^{'}(m) = e_3^{'}(m);$$

then, taking m=2 and introducing $e_{1 \text{ and } 3}^{'}(1)$ (using (28)) in place of $e_{1 \text{ and } 3}^{'}(2)$, we obtain (29) for m=1. The case m=2 in (29) is obtained in a similar manner. With (29), the elastic fields are :

$$u_3^{(0)(m)}(\vec{x}) = \frac{b}{2\pi}\tan^{-1}\frac{x_2}{x_1} + \frac{D_1 - D_2}{\mu_1 + \mu_2}\tan^{-1}\frac{x_1}{|x_2|},$$
$$\sigma_{23}^{(0)(m)}(\vec{x}) = \frac{b\mu_1\mu_2}{\pi(\mu_1 + \mu_2)}\frac{x_1}{r^2},$$
$$\sigma_{13}^{(0)(m)}(\vec{x}) = -\frac{b\mu_1\mu_2}{\pi(\mu_1 + \mu_2)}\frac{x_2}{r^2} + \delta_A(x_2)\frac{\mu_m(D_1 - D_2)}{\mu_1 + \mu_2}\pi\delta(x_1). \qquad (30)$$

These are in complete agreement with previous works (see [13] and references therein). Other elastic field components are zero.

III-2-2. Elastic fields of an interface sinusoidal screw dislocation

Five values for ($\bar{\alpha}_3^{(m)}(\kappa_n)$, $\bar{\beta}_3^{(m)}(\kappa_n)$) are extracted from (13); these are

$$\text{a)}\quad \overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1a}^{(m)}}{2}\frac{k_1}{\left(k_1^2+\kappa_n^2\right)^{3/2}} \equiv \overline{\alpha}_{3a}^{A_n(m)},\ \overline{\beta}_3^{(m)} = 0 \equiv \overline{\beta}_{3a}^{A_n(m)};$$

$$\text{b)}\quad \overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1b}^{(m)} k_1}{\sqrt{k_1^2+\kappa_n^2}}\left(\frac{4-\nu_m-3\rho_m}{k_1^2+\Omega_{1b}\kappa_n^2} - \frac{2(1-\rho_m)}{k_1^2+\kappa_n^2}\right) \equiv \overline{\alpha}_{3b}^{A_n(m)},$$

$$\overline{\beta}_3^{(m)} = \xi_n r_{1b}^{(m)} k_1\left(\frac{1}{k_1^2+\Omega_{1b}\kappa_n^2} - \frac{1}{k_1^2+\kappa_n^2}\right) \equiv \overline{\beta}_{3b}^{A_n(m)};$$

$$\text{c)}\quad \overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1c}^{(m)}}{\kappa_n^2}\left((2-\nu_m)(1+Q_r)\frac{\sqrt{k_1^2+\kappa_n^2}}{k_1} - \frac{2(2-\nu_m)\kappa_n^2}{k_1\sqrt{k_1^2+\kappa_n^2}} - \frac{2k_1}{\sqrt{k_1^2+\kappa_n^2}} + \frac{\kappa_n^2 k_1}{\left(k_1^2+\kappa_n^2\right)^{3/2}}\right) \equiv \overline{\alpha}_{3c}^{A_n(m)}$$

$$\overline{\beta}_3^{(m)} = \frac{\xi_n r_{1c}}{k_1}\left(1+Q_r - \frac{2\kappa_n^2}{k_1^2+\kappa_n^2}\right) \equiv \overline{\beta}_{3c}^{A_n} \text{ independent of } m;$$

$$\text{d)}\quad \overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1d}^{(m)} k_1}{\left(k_1^2+\kappa_n^2\right)^{3/2}}\left(s_{2d}^{(m)} + \frac{\kappa_n^2 s_{3d}^{(m)}}{k_1^2 - s_m\kappa_n^2} - \frac{\kappa_n^2 s_{4d}^{(m)}}{k_1^2 - r_m\kappa_n^2}\right) \equiv \overline{\alpha}_{3d}^{A_n(m)},$$

$$\overline{\beta}_3^{(m)} = \xi_n r_{1d}^{(m)} k_1\left(\frac{r_m+\tilde{Q}_r}{k_1^2 - r_m\kappa_n^2} + \frac{1-\tilde{Q}_r}{k_1^2+\kappa_n^2}\right) \equiv \overline{\beta}_{3d}^{A_n(m)};$$

$$\text{e)}\quad \overline{\alpha}_3^{(m)} = \frac{\xi_n s_{1e}^{(m)} k_1}{\sqrt{k_1^2+\kappa_n^2}}\left(\frac{s_{3e}^{(m)}}{k_1^2} + \frac{s_{2e}^{(m)} - s_{3e}^{(m)}}{k_1^2+\kappa_n^2}\right) \equiv \overline{\alpha}_{3e}^{A_n(m)},$$

$$\overline{\beta}_3^{(m)} = \xi_n r_{1e}^{(m)} k_1\left(-\frac{1}{k_1^2} + \frac{2}{k_1^2+\kappa_n^2}\right) \equiv \overline{\beta}_{3e}^{A_n(m)} \tag{31}$$

where

$s_{1a}^{(m)} = (-1)^m \mu_m b C_\nu / 8\pi$, $\quad C_\nu = \left[1/(1-\nu_1) - 1/(1-\nu_2)\right]$, $\quad Q_r = Q_c / Q_b = 1/\tilde{Q}_r$,

$r_m = \nu_m/(1-2\nu_m)$, $\quad \rho_m = \nu_1\delta_{m2} + \nu_2\delta_{m1}$, $\quad s_m = \rho_m/(1-2\rho_m)$,

$\Omega_{1b} = \left[(1-\nu_1)(1-2\nu_2) + (1-\nu_2)(1-2\nu_1)\right]/\ 4(1-\nu_1)(1-\nu_2)$;

$s_{1b}^{(m)} = (-1)^{m-1} C_\nu D_m / 8(2-\nu_1-\nu_2)$, $r_{1b}^{(m)} = C_\nu C_m / 16(1-\Omega_{1b})$;

$s_{1c}^{(m)} = (-1)^m i Q_b / 2$, $r_{1c} = -iQ_b/4$;

$s_{1d}^{(m)} = (-1)^m i Q_c / 2$,

$s_{2d}^{(m)} = -2\tilde{Q}_r + 2\left[\rho_m - \nu_m - 2\nu_m(1-2\rho_m)\right]/(1-2\nu_m)(1-2\rho_m)$,

$s_{3d}^{(m)} = (s_m + \tilde{Q}_r)/(1-2\rho_m)$, $s_{4d}^{(m)} = (r_m + \tilde{Q}_r)(5-4\nu_m)/(1-2\nu_m)$,

$$r_{1d}^{(m)} = iQ_c / \ 2(1-2\nu_m)(1+r_m)\,;$$
$$s_{1e}^{(m)} = (-1)^m iQ_b / \ 8(1-\nu_m)(1-\rho_m)\,, \qquad r_{1e}^{(m)} = i(Q_c - Q_b) / \ 4(1-\nu_m)\,,$$
$$s_{2e}^{(m)} = Q_r\left(-\rho_m - 3\nu_m + 4\nu_m\rho_m\right) - 4 + 5\rho_m + 7\nu_m - 8\nu_m\rho_m\,,$$
$$s_{3e}^{(m)} = (Q_r - 1)\left(8 - 7\rho_m - 5\nu_m + 4\nu_m\rho_m\right). \tag{32}$$

None of these couples satisfies the entire conditions (13). For each couple, we give in Appendix B the associated oscillating elastic fields $\vec{u}^{A_n(m)V} = \vec{u}^{A_n(m)A} + \vec{u}^{A_n(m)B}$ and $(\sigma)^{A_n(m)V} = (\sigma)^{A_n(m)A} + (\sigma)^{A_n(m)B}$ defined in (9). A superposition of these partial fields will provide the complete form of solution (to first order in ξ_n). The elastic fields $\vec{u}^{(m)}(\vec{x})$ and $(\sigma)^{(m)}(\vec{x})$ of an interface sinusoidal screw dislocation may be written as

$$\vec{u}^{(m)} = \vec{u}^{(0)(m)} + \vec{u}^{A_n(m)}$$
$$(\sigma)^{(m)} = (\sigma)^{(0)(m)} + (\sigma)^{A_n(m)}\,; \tag{33}$$

$\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$ (30) correspond to the fields of a straight screw dislocation lying on a planar interface; $\vec{u}^{A_n(m)}$ and $(\sigma)^{A_n(m)}$ are oscillating expressions proportional to either the sinusoid $A_n(x_3)$ or its spatial derivative $\partial A_n / \partial x_3$ in the forms

$$\begin{aligned}\vec{u}^{A_n(m)} &= \vec{u}^{A_n(m)\infty} - \vec{u}^{A_n(m)W} \\ (\sigma)^{A_n(m)} &= (\sigma)^{A_n(m)\infty} - (\sigma)^{A_n(m)W}\end{aligned}\,; \tag{34}$$

with

$$\vec{u}^{A_n(m)W} = \sum_{j=a \text{ to } e} \eta_j^{A_n(m)} \vec{u}_j^{A_n(m)V}$$
$$(\sigma)^{A_n(m)W} = \sum_{j=a \text{ to } e} \eta_j^{A_n(m)} (\sigma)_j^{A_n(m)V}\,. \tag{35}$$

$\vec{u}_{a \text{ to } e}^{A_n(m)V}$ and $(\sigma)_{a \text{ to } e}^{A_n(m)V}$ are given in Appendix B (for $\vec{u}^{A_n(m)\infty}$ and $(\sigma)^{A_n(m)\infty}$, see [5, 6]) ; $\eta_{a \text{ to } e}^{A_n(m)}$ are real to be determined by the requirement that the elastic fields be continuous when crossing the interface. It is sufficient to write this condition for points on the average interface plane. Before displaying the corresponding equations, we stress what follows.

- Both $u_1^{A_n(m)}(x_1, 0, x_3)$ and $\sigma_{13}^{A_n(m)}(x_1, 0, x_3)$ contain two terms with singularities $\ln|x_1|$ and $1/x_1^2$; the associated coefficients ($e_{11}^{A_n}$ and $e_{81}^{A_n}$)

and ($e_{12}^{A_n}$ and $e_{82}^{A_n}$), respectively, have been taken constant.

- $u_2^{A_n(m)}(x_1,0,x_3)$, $\sigma_{12}^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{23}^{A_n(m)}(x_1,0,x_3)$ are bounded functions. Under such conditions, we have considered their linear forms with respect to x_1 and posed the terms proportional to x_1 constant with m=1 and 2.
- $\sigma_{11}^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{22}^{A_n(m)}(x_1,0,x_3)$ have terms with singularities $1/x_1$ and those of $\partial^2 I_0/\partial|x_2|\partial x_1 \equiv \left(2/x_1^2+\kappa_n^2\ln\kappa_n|x_1|\right)/x_1$; the associated coefficients ($e_{41}^{A_n}$ and $e_{51}^{A_n}$) and ($e_{42}^{A_n}$ and $e_{52}^{A_n}$), respectively, have been set constant.
- $u_3^{A_n(m)}(x_1,0,x_3)$ and $\sigma_{33}^{A_n(m)}(x_1,0,x_3)$ contain terms with the singularity $1/x_1$ only. The associated coefficients $e_3^{A_n}$ and $e_6^{A_n}$ are taken constant.

We may write :

$$u_1^{A_n(1)}(x_1,0,x_3)=u_1^{A_n(2)}(x_1,0,x_3) \;\Rightarrow$$

$$\frac{1}{\mu_m}\left\{\frac{C_m(1-2\nu_m)}{4}-\eta_a^{A_n(m)}\frac{s_{1a}^{(m)}}{2}+\eta_b^{A_n(m)}\frac{s_{1a}^{(m)}}{2}+\eta_c^{A_n(m)}s_{1c}^{(m)}(1-2\nu_m)\right.$$
$$\left.-\eta_d^{A_n(m)}s_{1d}^{(m)}s_{2d}^{(m)}-\eta_e^{A_n(m)}s_{1e}^{(m)}s_{2e}^{(m)}\right\}\equiv e_{11}^{A_n},$$

$$\eta_c^{A_n(m)}\frac{1}{\mu_m}s_{1c}^{(m)}\left[(2-\nu_m)Q_r-\nu_m\right]\equiv e_{12}^{A_n};$$

$$u_2^{A_n(1)}=u_2^{A_n(2)} \;\Rightarrow$$

$$\frac{1}{\mu_m}\left\{\eta_a^{A_n(m)}(-1)^m s_{1a}^{(m)}/2-\eta_b^{A_n(m)}\left((-1)^m s_{1b}^{(m)}\left[2(1-\rho_m)-\sqrt{\Omega_{1b}}(4-\nu_m-3\rho_m)\right]\right.\right.$$
$$\left.+r_{1b}^{(m)}(1-\sqrt{\Omega_{1b}})\right)+\eta_d^{A_n(m)}\left((-1)^m s_{1d}^{(m)}\left[s_{2d}^{(m)}-\frac{s_{3d}^{(m)}}{1+s_m}+\frac{s_{4d}^{(m)}}{1+r_m}\right]+r_{1d}^{(m)}(1-\tilde{Q}_r)\right)$$
$$\left.+\eta_e^{A_n(m)}\left(2r_{1e}^{(m)}+(-1)^m s_{1e}^{(m)}\left[s_{2e}^{(m)}-s_{3e}^{(m)}\right]\right)\right\}\equiv e_2^{A_n};$$

$$u_3^{A_n(1)}=u_3^{A_n(2)} \;\Rightarrow$$

$$\frac{1}{\mu_m}\left\{D_m/2-\eta_c^{A_n(m)}s_{1c}^{(m)}\left[-2+\nu_m(1+Q_r)\right]+\eta_d^{A_n(m)}(-1)^{m-1}4(1-\nu_m)(1+r_m)r_{1d}^{(m)}\right.$$
$$\left.+\eta_e^{A_n(m)}(-1)^{m-1}4(1-\nu_m)r_{1e}^{(m)}\right\}\equiv e_3^{A_n};$$

$$\sigma_{11}^{A_n(1)} = \sigma_{11}^{A_n(2)} \;\Rightarrow$$

$$\frac{C_m}{4} - \eta_a^{A_n(m)} \frac{s_{1a}^{(m)}}{2} + \eta_b^{A_n(m)} \frac{s_{1a}^{(m)}}{2} + \eta_c^{A_n(m)} s_{1c}^{(m)} \left[1 - \nu_m (1 - Q_r)\right]$$
$$- \eta_d^{A_n(m)} \left[s_{1d}^{(m)} s_{2d}^{(m)} + (-1)^m 2\nu_m (1 + r_m) r_{1d}^{(m)}\right] - \eta_e^{A_n(m)} \left[s_{1e}^{(m)} s_{2e}^{(m)} + (-1)^m 2\nu_m r_{1e}^{(m)}\right] \equiv e_{41}^{A_n} ,$$
$$\eta_c^{A_n(m)} s_{1c}^{(m)} \left[(\nu_m - 2) Q_r + \nu_m\right] \equiv e_{42}^{A_n} ;$$

$$\sigma_{22}^{A_n(1)} = \sigma_{22}^{A_n(2)} \;\Rightarrow$$

$$\frac{C_m (1 - 2\nu_m)}{4} - \eta_a^{A_n(m)} \frac{s_{1a}^{(m)}}{2} + \eta_b^{A_n(m)} \frac{s_{1a}^{(m)}}{2} - \eta_c^{A_n(m)} s_{1c}^{(m)} \left[-2(1 - \nu_m) + Q_r\right]$$
$$- \eta_d^{A_n(m)} \left[s_{1d}^{(m)} s_{2d}^{(m)} + (-1)^m 2(1 - \nu_m)(1 + r_m) r_{1d}^{(m)}\right]$$
$$- \eta_e^{A_n(m)} \left[s_{1e}^{(m)} s_{2e}^{(m)} + (-1)^m 2(1 - \nu_m) r_{1e}^{(m)}\right] \equiv e_{51}^{A_n} ,$$
$$\eta_c^{A_n(m)} s_{1c}^{(m)} \left[(2 - \nu_m) Q_r - \nu_m\right] \equiv e_{52}^{A_n} ;$$

$$\sigma_{33}^{A_n(1)} = \sigma_{33}^{A_n(2)} \;\Rightarrow$$

$$\frac{C_m}{2} + \eta_c^{A_n(m)} 2 s_{1c}^{(m)} + \eta_d^{A_n(m)} (-1)^{m-1} 2(2 - \nu_m)(1 + r_m) r_{1d}^{(m)}$$
$$+ \eta_e^{A_n(m)} (-1)^{m-1} 2(2 - \nu_m) r_{1e}^{(m)} \equiv e_6^{A_n} ;$$

$$\sigma_{12}^{A_n(1)} = \sigma_{12}^{A_n(2)} \;\Rightarrow$$

$$\eta_a^{A_n(m)} (-1)^{m-1} \frac{s_{1a}^{(m)}}{2} + \eta_b^{A_n(m)} \Big((-1)^m s_{1b}^{(m)} \left[2(1 - \rho_m) - \Omega_{1b} (4 - \nu_m - 3\rho_m)\right]$$
$$+ r_{1b}^{(m)} (1 - \Omega_{1b}) \Big) + \eta_d^{A_n(m)} \Big((-1)^{m-1} s_{1d}^{(m)} \left[s_{2d}^{(m)} - s_{3d}^{(m)} + s_{4d}^{(m)}\right] + r_{1d}^{(m)} (1 + r_m)(r_m - 1 + \tilde{Q}_r) \Big)$$
$$+ \eta_e^{A_n(m)} \Big((-1)^m s_{1e}^{(m)} \left[s_{3e}^{(m)} - s_{2e}^{(m)}\right] - 2 r_{1e}^{(m)} \Big) \equiv e_7^{A_n} ;$$

$$\sigma_{13}^{A_n(1)} = \sigma_{13}^{A_n(2)} \;\Rightarrow$$

$$- \frac{C_m}{4} - \eta_a^{A_n(m)} \frac{s_{1a}^{(m)}}{2} - \eta_c^{A_n(m)} s_{1c}^{(m)} - \eta_e^{A_n(m)} \left[s_{1e}^{(m)} s_{2e}^{(m)} + (-1)^{m-1} 4(1 - \nu_m) r_{1e}^{(m)}\right]$$
$$- \eta_d^{A_n(m)} \left[s_{1d}^{(m)} s_{2d}^{(m)} + (-1)^{m-1} 2(1 - \nu_m)(1 + r_m)(1 - r_m - \tilde{Q}_r) r_{1d}^{(m)}\right] \equiv e_{81}^{A_n} ,$$
$$- \frac{D_m}{4} + \eta_c^{A_n(m)} s_{1c}^{(m)} (Q_r - 1) + \eta_d^{A_n(m)} (-1)^m 2(1 - \nu_m)(1 + r_m) r_{1d}^{(m)}$$
$$+ \eta_e^{A_n(m)} (-1)^m 2(1 - \nu_m) r_{1e}^{(m)} \equiv e_{82}^{A_n} ;$$

$$\sigma_{23}^{A_n(1)} = \sigma_{23}^{A_n(2)} \;\Rightarrow$$

$$\eta_a^{A_n(m)}(-1)^m \frac{s_{1a}^{(m)}}{2} - \eta_b^{A_n(m)}\Big((-1)^m s_{1b}^{(m)}\Big[2(1-\rho_m) - \sqrt{\Omega_{1b}}(4-\nu_m-3\rho_m)\Big] + r_{1b}^{(m)}\Big[1-\sqrt{\Omega_{1b}} - 2(1-\nu_m)(1-\Omega_{1b})\sqrt{\Omega_{1b}}\Big]\Big) + \eta_d^{A_n(m)}\left((-1)^m s_{1d}^{(m)}\left[s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1+s_m} + \frac{s_{4d}^{(m)}}{1+r_m}\right] + r_{1d}^{(m)}(1-\tilde{Q}_r)\right) + \eta_e^{A_n(m)}\Big((-1)^m s_{1e}^{(m)}\Big[s_{2e}^{(m)} - s_{3e}^{(m)}\Big] + 2r_{1e}^{(m)}\Big) \equiv e_9^{A_n}. \quad (36)$$

All $e_i^{A_n}(m)$ are constant with m=1 and 2 (i.e. $e_i^{A_n}(1) = e_i^{A_n}(2)$).We can see that $e_{42}^{A_n} = -e_{52}^{A_n} = -\mu_m e_{12}^{A_n}$. There are twelve equations in (36) with ten unknowns $\eta_{a \text{ to } e}^{A_n(m)}$. A solution can be found with ten independent equations. A methodology of the solution may be [11] : (*i*) express the $\eta_i^{A_n(2)}$ as a function of the $\eta_i^{A_n(1)}$ giving five relations, (*ii*) report these relations in five independent equations in (36); we have then a linear system of five equations with unknowns $\eta_i^{A_n(1)}$ that can be resolved by the usual classical method with determinants. The result is five expressions linking the $\eta_i^{A_n(1)}$ with the elastic constants of the mediums m=1 and 2. (*iii*) Come back to the $\eta_i^{A_n(2)}$ relations to find their respective values as a function of the elastic constants. Here however, we shall proceed differently by first calculating values of a number of $e_i^{A_n}$, including those associated with the singularity $1/x_1$ in the stress fields. When a crack is represented by a continuous distribution of infinitesimal dislocations, stress terms that have a singularity $1/x_1$ contribute a non-zero value to the crack extension force. We have, using (36),

$$\nu_m C_m/2 + (-1)^m 2(1-2\nu_m)\Big[\eta_c^{A_n(m)}(1+Q_r)r_{1c} + \eta_d^{A_n(m)}(1+r_m)r_{1d}^{(m)} + \eta_e^{A_n(m)}r_{1e}^{(m)}\Big] = e_{42}^{A_n} + e_{41}^{A_n} - e_{51}^{A_n} \equiv E_1^{A_n}. \quad (37)$$

Equations $e_3^{A_n}(1) = e_3^{A_n}(2)$, $e_6^{A_n}(1) = e_6^{A_n}(2)$, $E_1^{A_n}(1) = E_1^{A_n}(2)$ and $e_{42}^{A_n}(1) = e_{42}^{A_n}(2)$ are four relations that provide the values of unknowns $\eta_c^{A_n(m)}$ and $\eta_d^{A_n(m)}(1+r_m)r_{1d}^{(m)} + \eta_e^{A_n(m)}r_{1e}^{(m)}$, m=1 and 2, in the forms :

$$e_{52}^{A_n} = \eta_c^{A_n(m)} s_{1c}^{(m)}\big[(2-\nu_m)Q_r - \nu_m\big] = \frac{1}{3(\Gamma-1)(\nu_2-\nu_1)}\Big\{(\nu_2 C_2 - \nu_1 C_1)\big[(1-\nu_2)(2-\nu_1) - (1-\nu_1)(2-\nu_2)\Gamma\big]$$

$$+(C_2-C_1)\left[(1-\nu_2)(1-2\nu_1)-\Gamma(1-\nu_1)(1-2\nu_2)\right]\},$$

$$(-1)^m 4\left[\eta_d^{A_n(m)}(1+r_m)r_{1d}^{(m)}+\eta_e^{A_n(m)}r_{1e}^{(m)}\right]= \frac{(2-\rho_m)(\nu_2C_2-\nu_1C_1)+(1-2\rho_m)(C_2-C_1)}{3(\nu_2-\nu_1)}+2(1+Q_r)\eta_c^{A_n(m)}s_{1c}^{(m)}, \qquad (38)$$

where $\Gamma=\mu_2/\mu_1$. We also have (see (36))

$$\eta_c^{A_n(m)}s_{1c}^{(m)}(Q_r-1)+(-1)^m 2(1-\nu_m)\left[\eta_d^{A_n(m)}(1+r_m)r_{1d}^{(m)}+\eta_e^{A_n(m)}r_{1e}^{(m)}\right]=\mu_m e_{11}^{A_n}-e_{51}^{A_n}. \qquad (39)$$

(39) with m=1 and 2 is a system of two equations with unknowns $e_{11}^{A_n}$ and $e_{51}^{A_n}$ that can be solved with the help of the second of (38). A direct inspection of various $e_i^{A_n}$ in (36) shows that these can be given values with the help of (38). Hence, we display following expressions ($e_{52}^{A_n}$ is given in (38)) :

$$e_{11}^{A_n}=\frac{1}{6(\mu_2-\mu_1)(\nu_2-\nu_1)}\{(\nu_1C_1-\nu_2C_2)\left[(1-\nu_1)(2-\nu_2)-(1-\nu_2)(2-\nu_1)\right] +(C_1-C_2)\left[(1-\nu_1)(1-2\nu_2)-(1-\nu_2)(1-2\nu_1)\right]\},$$

$$e_{51}^{A_n}=-e_{52}^{A_n}/2,$$

$$e_3^{A_n}=\frac{b}{4\pi}+\frac{(1-\nu_1)(1-\nu_2)(\nu_1C_1-\nu_2C_2)}{\mu_1(1-\nu_2)(1-2\nu_1)-\mu_2(1-\nu_1)(1-2\nu_2)} +\frac{(1-\nu_1)(1-2\nu_2)-(1-\nu_2)(1-2\nu_1)}{\mu_1(1-\nu_2)(1-2\nu_1)-\mu_2(1-\nu_1)(1-2\nu_2)}e_{52}^{A_n},$$

$$e_{41}^{A_n}=\frac{\nu_1C_1(1-\nu_1)(1-2\nu_2)\Gamma-\nu_2C_2(1-\nu_2)(1-2\nu_1)}{2\left[\Gamma(1-\nu_1)(1-2\nu_2)-(1-\nu_2)(1-2\nu_1)\right]} +\frac{\nu_1(1-2\nu_2)\Gamma-\nu_2(1-2\nu_1)}{2\left[\Gamma(1-\nu_1)(1-2\nu_2)-(1-\nu_2)(1-2\nu_1)\right]}e_{52}^{A_n},$$

$$e_6^{A_n}=\frac{\Gamma C_1-C_2}{2(\Gamma-1)}+\frac{\Gamma(\nu_1C_1-\nu_2C_2)\left[(1-\nu_1)(2-\nu_2)-(1-\nu_2)(2-\nu_1)\right]}{2(\Gamma-1)\left[\Gamma(1-\nu_1)(1-2\nu_2)-(1-\nu_2)(1-2\nu_1)\right]} +\frac{\nu_1(1-2\nu_2)\Gamma-\nu_2(1-2\nu_1)}{2\left[\Gamma(1-\nu_1)(1-2\nu_2)-(1-\nu_2)(1-2\nu_1)\right]}e_{52}^{A_n}. \qquad (40)$$

These expressions are unchanged by inverting the elastic constants. At this stage, only $\eta_c^{A_n(m)}$ is given a value (38). To proceed further, we introduce two parameters

$$\eta_{b+a}^{A_n(m)} = \eta_b^{A_n(m)} + \eta_a^{A_n(m)},$$
$$\eta_{b-a}^{A_n(m)} = \eta_b^{A_n(m)} - \eta_a^{A_n(m)} \tag{41}$$

and search for these two parameters and $\eta_d^{A_n(m)}$ and $\eta_e^{A_n(m)}$, m=1 and 2; we have eight unknowns and look for eight independent equations. Because the value of $e_{11}^{A_n}$ is known (40), its expressions with the associated $\eta_i^{A_n(m)}$ (36) provide two equations; two additional relations are given by the second of (38) with $\eta_d^{A_n(m)}(1+r_m)r_{1d}^{(m)} + \eta_e^{A_n(m)} r_{1e}^{(m)}$. These first four equations can be written in the form

$$c_{11}^{(m)}\eta_d^{A_n(m)} + c_{12}^{(m)}\eta_e^{A_n(m)} = b_1^{(m)},$$
$$c_{21}^{(m)}\eta_d^{A_n(m)} + c_{22}^{(m)}\eta_e^{A_n(m)} = b_2^{(m)} \tag{42}$$

where

$$c_{11}^{(m)} = -s_{1d}^{(m)} s_{2d}^{(m)},\ c_{12}^{(m)} = -s_{1e}^{(m)} s_{2e}^{(m)},$$
$$b_1^{(m)} = -\eta_{b-a}^{A_n(m)} \frac{s_{1a}^{(m)}}{2} + \mu_m e_{11}^{A_n} - \frac{C_m(1-2\nu_m)}{4} - \eta_c^{A_n(m)} s_{1c}^{(m)}(1-2\nu_m),$$
$$c_{21}^{(m)} = (1+r_m) r_{1d}^{(m)},\ c_{22}^{(m)} = r_{1e}^{(m)},$$
$$b_2^{(m)} = \frac{(-1)^m}{4}\left(\frac{(2-\rho_m)(\nu_2 C_2 - \nu_1 C_1) + (1-2\rho_m)(C_2 - C_1)}{3(\nu_2 - \nu_1)} \right.$$
$$\left. + 2(1+Q_r)\eta_c^{A_n(m)} s_{1c}^{(m)} \right).$$

We obtain

$$\eta_e^{A_n(m)} = \frac{c_{21}^{(m)} b_1^{(m)} - c_{11}^{(m)} b_2^{(m)}}{c_{21}^{(m)} c_{12}^{(m)} - c_{11}^{(m)} c_{22}^{(m)}},$$
$$\eta_d^{A_n(m)} = \frac{c_{22}^{(m)} b_1^{(m)} - c_{12}^{(m)} b_2^{(m)}}{c_{22}^{(m)} c_{11}^{(m)} - c_{12}^{(m)} c_{21}^{(m)}}. \tag{43}$$

Relations (43) give $\eta_d^{A_n(m)}$ and $\eta_e^{A_n(m)}$, m=1 and 2, as a function of $\eta_{b-a}^{A_n(m)}$ only through $b_1^{(m)}$. It remains to evaluate $\eta_{b+a}^{A_n(m)}$ and $\eta_{b-a}^{A_n(m)}$ with four independent equations. We may use : $e_2^{A_n}(1) = e_2^{A_n}(2)$, $e_7^{A_n}(1) = e_7^{A_n}(2)$, $e_9^{A_n}(1) + e_7^{A_n}(1) = e_9^{A_n}(2) + e_7^{A_n}(2)$ and $e_{81}^{A_n}(1) = e_{81}^{A_n}(2)$. We then have four independent equations with unknown $\eta_{b\pm a}^{A_n(m)}$ that can be resolved by the usual classical method with determinants (see [11] for analogous analysis). In summary, the elastic fields ($\vec{u}^{(m)}$ and $(\sigma)^{(m)}$: (33)) of an interfacial sinusoidal screw dislocation ***(Figure 2)*** to linear order in ξ_n, are the sum of two terms: the first ones correspond to the elastic fields ($\vec{u}^{(0)(m)}$ and $(\sigma)^{(0)(m)}$: (30)) produced by an interface straight screw dislocation; second terms are oscillating expressions ($\vec{u}^{A_n(m)}, (\sigma)^{A_n(m)}$: (34)), proportional to the perturbation $A_n(x_3) = \xi_n \sin \kappa_n x_3$ or its partial derivative $\partial A_n / \partial x_3$. These latter expressions can be written as linear combinations (35) of partial elastic fields $\vec{u}_i^{A_n(m)V}$ (*i*= *a* to *e*) (see Appendix B). The associated proportionality coefficients $\eta_i^{A_n(m)}$ fulfilled continuity requirements (36) of the elastic ($\vec{u}^{(m)}, (\sigma)^{(m)}$) on crossing the sinusoidal interface. These can be solved to provide values for the $\eta_i^{A_n(m)}$ (see (38) and (43) with the associated text). A number of $e_i^{A_n}$ values have been calculated (40) that include the coefficients of the various stress terms with the singularity $1/x_1$. We shall make use of certain terms in the crack analysis to come (part II of this study).

IV - DISCUSSION AND CONCLUDING REMARKS

In the present study, the displacement and stress fields of sinusoidal dislocations (glide-type edge and screw) lying, at the origin, on a non-planar interface *S* with the form of a corrugated sheet ***(Figure 2)***, have been determined. Stress terms with the singularity $1/x_1$ have been emphasized in view of crack analyses. As an illustration, consider the stress $\sigma_{33}^{(m)}$ (case of sinusoidal screw dislocation only): on the interface $x_2 = A_n$ is assumed small. We can take the linear form of the stress up to term with x_2, i.e.

$$\sigma_{33}^{(m)}(x_1, x_2, x_3) = \sigma_{33}^{(m)}(x_1, 0, x_3) + \frac{\partial \sigma_{33}^{(m)}(x_1, 0, x_3)}{\partial x_2} x_2 .$$

We have

$$\sigma_{33}^{(m)}(x_1, 0, x_3) = \sigma_{33}^{(0)(m)}(x_1, 0, x_3) + \sigma_{33}^{A_n(m)}(x_1, 0, x_3) .$$

The first term $\sigma_{33}^{(0)(m)}$ (30) is zero. Restricting ourselves to stress terms with $1/x_1$ only, it is easy to see that

$$\sigma_{33}^{A_n(m)}(x_1, 0, x_3) = -\frac{\partial A_n}{\partial x_3} \frac{4 e_6^{A_n}}{x_1} . \tag{44}$$

The value of $e_6^{A_n}$ is known (40) with no reference to the $\eta_i^{A_n(m)}$. This result is sufficient in the analysis of a number of special crack fronts when $\sigma_{33}^{(m)}$ is involved (see [5, 6] for the case of homogeneous solids). But in general, it will be necessary to write down explicitly the value of all the $\eta_i^{A_n(m)}$ in order to have the stress singularities. This study also reveals the presence of a term with Dirac delta function $\delta(x_1)$ in $\sigma_{13}^{(0)(m)}$ (30) on the interface at $x_2 = 0$. The associated coefficient is proportional to shear modulus μ_m; hence it changes with m=1 and 2. No alternative has been found to this value. Assume now that the interface with the associated dislocation have a more general form f (1) in the x_2x_3-plane. Writing $B_n = \xi_n \sin\kappa_n x_3 + \delta_n \cos\kappa_n x_3$ and assuming ξ small, the elastic fields $\vec{u}^{(m)}$ and $(\sigma)^{(m)}$ in the bi-material are simply (to linear terms in the amplitudes)

$$\vec{u}^{(m)} = \vec{u}_h^{(0)(m)} + \vec{u}_\xi^{(m)},$$
$$(\sigma)^{(m)} = (\sigma)_h^{(0)(m)} + (\sigma)_\xi^{(m)}; \tag{45}$$

where

$$\vec{u}_\xi^{(m)} = \sum_n \vec{u}^{B_n(m)},$$
$$(\sigma)_\xi^{(m)} = \sum_n (\sigma)^{B_n(m)}. \tag{46}$$

Here, $\vec{u}_h^{(0)(m)}$ and $(\sigma)_h^{(0)(m)}$ are the fields of an interface straight dislocation, displaced by $x_2 = h$ from the origin. In the various elastic field expressions obtained with sinusoidal dislocations with shape A_n, we just have to replace A_n with B_n and add the symbol $\sum$ in front of the oscillating elastic fields; in addition, x_2 has to be replaced by $x_2 - h$. We shall proceed further by providing expressions for the elastic fields of interface climb-type edge sinusoidal dislocation ($\vec{b}_{II} = (b,0,0)$), crack tip stresses and crack extension force. These will be the subject of part II of the work.

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APPENDIX A : DIFFERENCE OF THE VALUES OF $\vec{u}^{(m)\infty}$ AND $(\sigma)^{(m)\infty}$ (SCREW) WHEN CROSSING THE INTERFACE

Our purpose here is to write down the differences $\Delta\vec{u}^{\infty}$ and $(\Delta\sigma)^{\infty}$ (4) on crossing the interface at arbitrary point $P_S(x_1, x_2 = \xi_n \sin\kappa_n x_3, x_3)$. We use the notation $x_2 = \xi$ ($\xi = \xi_n \sin\kappa_n x_3$ small) and take the MacLaurin series expansions of the elastic fields up to terms of first order with respect to ξ; this means that

$$\Delta\vec{u}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta\vec{u}^{\infty}(x_1, 0, x_3) + \frac{\partial\Delta\vec{u}^{\infty}}{\partial x_2}(x_1, 0, x_3)\xi\,,$$

$$(\Delta\sigma)^{\infty}(x_1, x_2 = \xi, x_3) = (\Delta\sigma)^{\infty}(x_1, 0, x_3) + \frac{\partial(\Delta\sigma)^{\infty}}{\partial x_2}(x_1, 0, x_3)\xi\,. \qquad \text{(A.1)}$$

$\vec{u}^{(m)\infty}$ and $(\sigma)^{(m)\infty}$ are taken from our previous works [5, 6]. We obtain (u_i is the *i*-component of vector $\vec{u}$ and σ_{ij} the *ij*-element of the stress matrix (σ); *i*, *j*= 1 to 3)

$$\Delta u_i^{\infty}(x_1, x_2 = \xi, x_3) = \Delta u_i^{(0)\infty} + \Delta u_i^{A_n\infty}$$
$$\Delta\sigma_{ij}^{\infty}(x_1, x_2 = \xi, x_3) = \Delta\sigma_{ij}^{(0)\infty} + \Delta\sigma_{ij}^{A_n\infty}$$

as

$$\Delta u_1^{(0)\infty} = 0\,,$$

$$\Delta u_1^{A_n\infty} = \frac{\partial A_n}{\partial x_3}\frac{bC_\nu}{8\pi}\int_{-\infty}^{\infty}\frac{k_1^2}{\left(k_1^2+\kappa_n^2\right)^{3/2}}e^{ik_1x_1}dk_1;$$

$$\Delta u_2^{(0)\infty} = 0,$$

$$\Delta u_2^{A_n\infty} = \frac{\partial A_n}{\partial x_3}\frac{bC_\nu}{8\pi}\int_{-\infty}^{\infty}\frac{ik_1}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1\xi;$$

$$\Delta u_3^{(0)\infty} = 0,$$

$$\Delta u_3^{A_n\infty} = \kappa_n^2 A_n\frac{bC_\nu}{8\pi}\int_{-\infty}^{\infty}\frac{ik_1}{\left(k_1^2+\kappa_n^2\right)^{3/2}}e^{ik_1x_1}dk_1;$$

$$\Delta\sigma_{11}^{(0)\infty} = 0 \ ,$$

$$\Delta\sigma_{11}^{A_n\infty} = 2Q_b\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{k_1(k_1^2+2\kappa_n^2)}{\left(k_1^2+\kappa_n^2\right)^{3/2}}e^{ik_1x_1}dk_1;$$

$$\Delta\sigma_{22}^{(0)\infty} = 0,$$

$$\Delta\sigma_{22}^{A_n\infty} = 2(2Q_c-Q_b)\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{k_1}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1;$$

$$\Delta\sigma_{33}^{(0)\infty} = 0,$$

$$\Delta\sigma_{33}^{A_n\infty} = 2Q_b\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{k_1(2k_1^2+\kappa_n^2)}{\left(k_1^2+\kappa_n^2\right)^{3/2}}e^{ik_1x_1}dk_1;$$

$$\Delta\sigma_{12}^{(0)\infty} = 0,$$

$$\Delta\sigma_{12}^{A_n\infty} = 2i\frac{\partial A_n}{\partial x_3}\int_{-\infty}^{\infty}\frac{Q_b\kappa_n^2+(Q_c-2Q_b)(k_1^2+\kappa_n^2)}{\sqrt{k_1^2+\kappa_n^2}}e^{ik_1x_1}dk_1\xi;$$

$$\Delta\sigma_{13}^{(0)\infty} = 2i(Q_c-Q_b)\int_{-\infty}^{\infty}|k_1|e^{ik_1x_1}dk_1\xi,$$

$$\Delta\sigma_{13}^{A_n\infty} = -2iA_n\int_{-\infty}^{\infty}\frac{Q_b\kappa_n^2k_1^2+(Q_c-Q_b)(k_1^2+\kappa_n^2)^2}{\left(k_1^2+\kappa_n^2\right)^{3/2}}e^{ik_1x_1}dk_1;$$

$$\Delta\sigma_{23}^{(0)\infty} = 2(Q_c - Q_b)\int_{-\infty}^{\infty} \mathrm{sgn}(k_1) e^{ik_1 x_1} dk_1 ,$$

$$\Delta\sigma_{23}^{A_n\infty} = 2A_n \int_{-\infty}^{\infty} \frac{k_1\left[(Q_b - Q_c)k_1^2 + (2Q_b - Q_c)\kappa_n^2\right]}{\left(k_1^2 + \kappa_n^2\right)^{1/2}} e^{ik_1 x_1} dk_1 \xi \; ; \qquad \text{(A.2)}$$

where

$$C_\nu = [1/(1-\nu_1) - 1/(1-\nu_2)], \quad Q_b = i(C_2 - C_1)/4,$$
$$Q_c = i(\nu_2 C_2 - \nu_1 C_1)/4, \quad C_m = b\mu_m / 2\pi(1-\nu_m);$$

$\mathrm{sgn}(k_1) = k_1/|k_1|$; μ_m and ν_m are shear modulus and Poisson's ratio.

APPENDIX B : PARTIAL OSCILLATING ELASTIC FIELDS (SCREW)

The couple $(\overline{\alpha}_{3a}^{A_n(m)}, \overline{\beta}_{3a}^{A_n(m)})$ is obtained from (13 *a*, *b*, *d* and *e*) associated with the displacement. We have at position $\vec{x} = (x_1, x_2, x_3)$ $(\vec{u}^{A_n(m)V} \equiv \vec{u}_a^{A_n(m)V}$, $(\sigma)^{A_n(m)V} \equiv (\sigma)_a^{A_n(m)V})$:

$$u_{ia}^{A_n(m)V} = \frac{s_{1a}^{(m)}}{2\mu_m}\left\|(\delta_{i1}+\delta_{i2})\frac{\partial}{\partial x_3} + \delta_{i3}\right\| A_n \left(2K_0[\kappa_n r]\delta_{i1} + (-1)^{m-1}\frac{\partial \Pi_1}{\partial x_1} + \kappa_n^2 \left\| -\delta_{i1} + \delta_{i3}\frac{\partial}{\partial x_1}\right\| I_{1a} \right),$$

$$\sigma_{iia}^{A_n(m)V} = \frac{\partial A_n}{\partial x_3}\kappa_n s_{1a}^{(m)}\left((-\delta_{i1}+\delta_{i2})\frac{2x_1 K_1[\kappa_n r]}{r} + \kappa_n(-\delta_{i1}+\delta_{i3})\frac{\partial I_{1a}}{\partial x_1}\right),$$

$$\sigma_{12a}^{A_n(m)V} = \frac{\partial A_n}{\partial x_3}\kappa_n s_{1a}^{(m)}(-1)^m\left(\frac{2|x_2|K_1[\kappa_n r]}{r} - \kappa_n \Pi_1\right),$$

$$\sigma_{j3a}^{A_n(m)V} = A_n \kappa_n^2 s_{1a}^{(m)}\left(\delta_{j1}\left(\kappa_n^2 I_{1a} - 2K_0[\kappa_n r]\right) + \delta_{j2}(-1)^m \frac{\partial \Pi_1}{\partial x_1}\right); \qquad \text{(B.1)}$$

$$\Pi_z = \int_{-\infty}^{\infty} \frac{e^{(-1)^m\sqrt{k_1^2+\kappa_n^2}\,x_2}}{k_1^2 + z\kappa_n^2} e^{ik_1x_1} dk_1 \equiv \int_{-\infty}^{\infty} \frac{e^{-\sqrt{k_1^2+\kappa_n^2}\,|x_2|}}{k_1^2 + z\kappa_n^2} e^{ik_1x_1} dk_1 ,$$

$$I_{1a} = \int_{-\infty}^{\infty} \frac{e^{(-1)^m\sqrt{k_1^2+\kappa_n^2}\,x_2}}{\left(k_1^2+\kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 \equiv \int_{-\infty}^{\infty} \frac{e^{-\sqrt{k_1^2+\kappa_n^2}\,|x_2|}}{\left(k_1^2+\kappa_n^2\right)^{3/2}} e^{ik_1x_1} dk_1 .$$

Terms in brackets $\|\ \|$ are operators acting on A_n and I_{1a}, separately; Π_1 is the value of Π_z for z=1, $r^2 = x_1^2 + x_2^2$ and subscripts i=1 to 3 and j=1 and 2; $K_n[x]$ is the nth-order modified Bessel function usually so denoted and δ_{ij} is the Kronecker delta. We stress that the various integrations (such as in I_{1a} and Π_z) performed in the present study are given for spatial positions satisfying the condition $(-1)^m x_2 < 0$ (*i.e.* $(-1)^{m-1} = \mathrm{sgn}(x_2)$) with $m=1$ when $x_2 > \xi_n \sin\kappa_n x_3$ (half-space 1) and $m=2$ when $x_2 < \xi_n \sin\kappa_n x_3$ (half-space 2). However, this makes no difference in the elastic fields to first order in ξ_n.

The pair $(\overline{\alpha}_{3b}^{A_n(m)}, \overline{\beta}_{3b}^{A_n(m)})$ is obtained using (13 *b* to *e*) associated with the displacement. We have (using the similar notations)

$$u_{1b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}\frac{s_{1b}^{(m)}}{\mu_m}\left(\frac{\nu_1+\nu_2-2-2\Omega_{1b}(1-\rho_m)}{1-\Omega_{1b}}\frac{\partial \Pi_1}{\partial |x_2|} + \frac{\Omega_{1b}(4-\nu_m-3\rho_m)}{1-\Omega_{1b}}\frac{\partial \Pi_{\Omega_{1b}}}{\partial |x_2|} + 2(1-\rho_m)\kappa_n^2 I_{1a}\right),$$

$$u_{1b}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3}\frac{r_{1b}^{(m)}}{\mu_m}\kappa_n^2 x_2\left(\Pi_1 - \Omega_{1b}\Pi_{\Omega_{1b}}\right);$$

$$u_{2b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}\frac{s_{1b}^{(m)}}{\mu_m}(-1)^{m-1}\frac{\partial}{\partial x_1}\left([4-\nu_m-3\rho_m]\Pi_{\Omega_{1b}} - 2[1-\rho_m]\Pi_1\right),$$

$$u_{2b}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3}\frac{r_{1b}^{(m)}}{\mu_m}\left\|\frac{\partial}{\partial x_1} + (-1)^{m-1}x_2\frac{\partial^2}{\partial x_1 \partial |x_2|}\right\|(\Pi_1 - \Pi_{\Omega_{1b}});$$

$$u_{3b}^{A_n(m)A} = A_n\frac{s_{1b}^{(m)}}{\mu_m}\left(2\kappa_n^2(\rho_m - 1)\frac{\partial I_{1a}}{\partial x_1} + \frac{4-\nu_m-3\rho_m}{1-\Omega_{1b}}\frac{\partial^2}{\partial x_1 \partial |x_2|}(\Pi_1 - \Pi_{\Omega_{1b}})\right),$$

$$u_{3b}^{A_n(m)B} = A_n\frac{r_{1b}^{(m)}}{\mu_m}\left\|-\kappa_n^2 x_2\frac{\partial}{\partial x_1} + (-1)^m 4(1-\nu_m)\frac{\partial^2}{\partial x_1 \partial |x_2|}\right\|(\Pi_1 - \Pi_{\Omega_{1b}});$$

$$\sigma_{11b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}2s_{1b}^{(m)}\frac{\partial}{\partial x_1}\left(\frac{\partial \Pi_1}{\partial |x_2|}\frac{\nu_1+\nu_2-2-2\Omega_{1b}(1-\rho_m)}{1-\Omega_{1b}} + \frac{\partial \Pi_{\Omega_{1b}}}{\partial |x_2|}\frac{\Omega_{1b}(4-\nu_m-3\rho_m)}{1-\Omega_{1b}} + 2(1-\rho_m)\kappa_n^2 I_{1a}\right),$$

$$\sigma_{11b}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1b}^{(m)} \frac{\partial}{\partial x_1}\left(\kappa_n^2 x_2 (\Pi_1 - \Omega_{1b}\Pi_{\Omega_{1b}}) + (-1)^m 2\nu_m \frac{\partial}{\partial |x_2|}(\Pi_1 - \Pi_{\Omega_{1b}}) \right);$$

$$\sigma_{22b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1b}^{(m)} \frac{\partial^2}{\partial x_1 \partial |x_2|}\left([4 - \nu_m - 3\rho_m]\Pi_{\Omega_{1b}} - 2[1-\rho_m]\Pi_1 \right),$$

$$\sigma_{22b}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3} 2r_{1b}^{(m)} \frac{\partial}{\partial x_1}\left((1-\Omega_{1b})\kappa_n^2 x_2 \Pi_{\Omega_{1b}} + (-1)^m 2(1-\nu_m)\frac{\partial}{\partial |x_2|}(\Pi_1 - \Pi_{\Omega_{1b}}) \right);$$

$$\sigma_{33b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1b}^{(m)} \left(2\kappa_n^2(\rho_m - 1)\frac{\partial I_{1a}}{\partial x_1} + \frac{4 - \nu_m - 3\rho_m}{1 - \Omega_{1b}} \frac{\partial^2}{\partial x_1 \partial |x_2|}(\Pi_1 - \Pi_{\Omega_{1b}}) \right),$$

$$\sigma_{33b}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1b}^{(m)} \left\| -\kappa_n^2 x_2 \frac{\partial}{\partial x_1} + (-1)^m 2(2-\nu_m)\frac{\partial^2}{\partial x_1 \partial |x_2|} \right\| (\Pi_1 - \Pi_{\Omega_{1b}});$$

$$\sigma_{12b}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1b}^{(m)} (-1)^m \Big(I_0 (2 - \nu_1 - \nu_2)$$
$$+ 2(1-\rho_m)\kappa_n^2 \Pi_1 - [4 - \nu_m - 3\rho_m]\Omega_{1b}\kappa_n^2 \Pi_{\Omega_{1b}} \Big),$$

$$\sigma_{12b}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1b}^{(m)} \kappa_n^2 \left\| 1 + (-1)^{m-1} x_2 \frac{\partial}{\partial |x_2|} \right\| (\Pi_1 - \Omega_{1b}\Pi_{\Omega_{1b}});$$

$$\sigma_{13b}^{A_n(m)A} = -A_n 2s_{1b}^{(m)} \kappa_n^2 \Bigg(\frac{\partial \Pi_1}{\partial |x_2|} \frac{\nu_1 + \nu_2 - 2 - 2\Omega_{1b}(1-\rho_m)}{1 - \Omega_{1b}}$$
$$+ \frac{\partial \Pi_{\Omega_{1b}}}{\partial |x_2|} \frac{\Omega_{1b}(4 - \nu_m - 3\rho_m)}{1 - \Omega_{1b}} + 2(1-\rho_m)\kappa_n^2 I_{1a} \Bigg),$$

$$\sigma_{13b}^{A_n(m)B} = -A_n r_{1b}^{(m)} \kappa_n^2 \left\| 2\kappa_n^2 x_2 + (-1)^{m-1} 4(1-\nu_m)\frac{\partial}{\partial |x_2|} \right\| (\Pi_1 - \Omega_{1b}\Pi_{\Omega_{1b}});$$

$$\sigma_{23b}^{A_n(m)A} = A_n 2s_{1b}^{(m)} (-1)^m \kappa_n^2 \frac{\partial}{\partial x_1}\left([4 - \nu_m - 3\rho_m]\Pi_{\Omega_{1b}} - 2[1-\rho_m]\Pi_1 \right),$$

$$\sigma_{23b}^{A_n(m)B} = A_n 2r_{1b}^{(m)} \kappa_n^2 \frac{\partial}{\partial x_1}\Big\{ [1 + 2(1-\nu_m)(1-\Omega_{1b})]\Pi_{\Omega_{1b}} - \Pi_1$$

$$+(-1)^m x_2 \frac{\partial}{\partial|x_2|}(\Pi_1 - \Pi_{\Omega_{1b}})\Bigg\}; \qquad \text{(B.2)}$$

$$I_0 = \int_{-\infty}^{\infty} e^{(-1)^m\sqrt{k_1^2+\kappa_n^2}x_2} e^{ik_1x_1} dk_1 \equiv \int_{-\infty}^{\infty} e^{-\sqrt{k_1^2+\kappa_n^2}|x_2|} e^{ik_1x_1} dk_1 = \frac{2\kappa_n|x_2|}{r} K_1 .$$

The couple $(\overline{\alpha}_{3c}^{A_n(m)}, \overline{\beta}_{3c}^{A_n(m)})$ is obtained using (13 *j*, *k*, *n* and *o*) associated with stresses. We obtain

$$u_{1c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}\frac{s_{1c}^{(m)}}{\mu_m}\frac{1}{\kappa_n^2}\Bigg((\nu_m - (2-\nu_m)Q_r)\frac{\partial I_0}{\partial|x_2|}$$

$$-2(1-2\nu_m)\kappa_n^2 K_0[\kappa_n r] - \kappa_n^4 I_{1a}\Bigg),$$

$$u_{1c}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3}\frac{r_{1c}}{\mu_m} x_2\big((1+Q_r)I_0 - 2\kappa_n^2\Pi_1\big);$$

$$u_{2c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}\frac{s_{1c}^{(m)}}{\mu_m}\frac{1}{\kappa_n^2}(-1)^{m-1}\Bigg((-\nu_m + (2-\nu_m)Q_r)\frac{\partial I_0}{\partial x_1}$$

$$+\kappa_n^2\frac{\partial \Pi_1}{\partial x_1} + (2-\nu_m)(Q_r - 1)\kappa_n^2 i I_{1c}\Bigg),$$

$$u_{2c}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3}\frac{r_{1c}}{\mu_m}\left\|1 + (-1)^{m-1}x_2\frac{\partial}{\partial|x_2|}\right\|\left(2\frac{\partial \Pi_1}{\partial x_1} + i(Q_r - 1)I_{1c}\right);$$

$$u_{3c}^{A_n(m)A} = A_n\frac{s_{1c}^{(m)}}{\mu_m}\left(-2\nu_m\frac{\partial K_0}{\partial x_1} + i(2-\nu_m)(1-Q_r)\frac{\partial I_{1c}}{\partial|x_2|} + \kappa_n^2\frac{\partial I_{1a}}{\partial x_1}\right),$$

$$u_{3c}^{A_n(m)B} = A_n\frac{r_{1c}}{\mu_m}\left\|\kappa_n^2 x_2 + (-1)^{m-1}4(1-\nu_m)\frac{\partial}{\partial|x_2|}\right\|\left(2\frac{\partial \Pi_1}{\partial x_1} + i(Q_r - 1)I_{1c}\right);$$

$$\sigma_{11c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}2s_{1c}^{(m)}\frac{1}{\kappa_n^2}\Bigg(\frac{\partial^2 I_0}{\partial x_1\partial|x_2|}\big(\nu_m + Q_r(\nu_m - 2)\big) - \kappa_n^4\frac{\partial I_{1a}}{\partial x_1}$$

$$-2\kappa_n^2(1-2\nu_m)\frac{\partial K_0}{\partial x_1}\Bigg),$$

$$\sigma_{11c}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1c}\left(x_2 \frac{\partial}{\partial x_1}\left[(1+Q_r)I_0 - 2\kappa_n^2\Pi_1\right] + (-1)^{m-1}2\nu_m \frac{\partial}{\partial |x_2|}\left[2\frac{\partial \Pi_1}{\partial x_1} + i(Q_r-1)I_{1c}\right]\right);$$

$$\sigma_{22c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1c}^{(m)} \frac{1}{\kappa_n^2}\left(\frac{\partial^2 I_0}{\partial x_1 \partial |x_2|}\left(Q_r(2-\nu_m)-\nu_m\right) - 2\kappa_n^2 \frac{\partial K_0}{\partial x_1} + i\kappa_n^2(2-\nu_m)(Q_r-1)\frac{\partial I_{1c}}{\partial |x_2|}\right),$$

$$\sigma_{22c}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3} 2r_{1c}\left(x_2\left[(1+Q_r)\frac{\partial I_0}{\partial x_1} + i\kappa_n^2(Q_r-1)I_{1c}\right] + (-1)^m 2(1-\nu_m)\left[4\frac{\partial K_0}{\partial x_1} + i(1-Q_r)\frac{\partial I_{1c}}{\partial |x_2|}\right]\right);$$

$$\sigma_{33c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1c}^{(m)}\left(4(1-\nu_m)\frac{\partial K_0}{\partial x_1} + i(2-\nu_m)(1-Q_r)\frac{\partial I_{1c}}{\partial |x_2|} + \kappa_n^2 \frac{\partial I_{1a}}{\partial x_1}\right),$$

$$\sigma_{33c}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1c}\left(x_2\kappa_n^2\left[2\frac{\partial \Pi_1}{\partial x_1} + i(Q_r-1)I_{1c}\right] + (-1)^m 2(2-\nu_m)\left[4\frac{\partial K_0}{\partial x_1} + i(1-Q_r)\frac{\partial I_{1c}}{\partial |x_2|}\right]\right);$$

$$\sigma_{12c}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1c}^{(m)} \frac{1}{\kappa_n^2}(-1)^m\left(\frac{\partial^2 I_0}{\partial x_1^2}\left(\nu_m + Q_r(\nu_m-2)\right) - \kappa_n^4\Pi_1 + \kappa_n^2\left[(2-\nu_m)Q_r + \nu_m - 1\right]I_0\right),$$

$$\sigma_{12c}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1c}\left((1+Q_r)I_0 - 2\kappa_n^2\Pi_1 + (-1)^{m-1}x_2 \frac{\partial}{\partial |x_2|}\left[(1+Q_r)I_0 - 2\kappa_n^2\Pi_1\right]\right);$$

$$\sigma_{13c}^{A_n(m)A} = -A_n 2s_{1c}^{(m)}\left((\nu_m - (2-\nu_m)Q_r)\frac{\partial I_0}{\partial |x_2|} - 2(1-2\nu_m)\kappa_n^2 K_0[\kappa_n r] - \kappa_n^4 I_{1a} \right),$$

$$\sigma_{13c}^{A_n(m)B} = -A_n r_{1c}\left((-1)^{m-1} 4(1-\nu_m)\frac{\partial}{\partial |x_2|}\left[(1+Q_r)I_0 - 2\kappa_n^2 \Pi_1\right] + x_2 2\kappa_n^2 \left[(1+Q_r)I_0 - 2\kappa_n^2 \Pi_1 \right]\right);$$

$$\sigma_{23c}^{A_n(m)A} = A_n 2s_{1c}^{(m)}(-1)^m \left(\left[(2-\nu_m)Q_r - \nu_m\right]\frac{\partial I_0}{\partial x_1} + i\kappa_n^2(2-\nu_m)(Q_r-1)I_{1c} + \kappa_n^2 \frac{\partial \Pi_1}{\partial x_1}\right),$$

$$\sigma_{23c}^{A_n(m)B} = A_n r_{1c}\left(i2\kappa_n^2(3-2\nu_m)(Q_r-1)I_{1c} + 4\kappa_n^2\frac{\partial \Pi_1}{\partial x_1} + 4(1-\nu_m)(1+Q_r)\frac{\partial I_0}{\partial x_1} + x_2 2\kappa_n^2(-1)^m\left[4\frac{\partial K_0}{\partial x_1} + i(1-Q_r)\frac{\partial I_{1c}}{\partial |x_2|}\right]\right); \qquad \text{(B.3)}$$

$$I_{1c} \equiv \int_{-\infty}^{\infty} \frac{e^{-\sqrt{k_1^2+\kappa_n^2}|x_2|}}{k_1} e^{ik_1x_1} dk_1 .$$

The pair $(\overline{\alpha}_{3d}^{A_n(m)}, \overline{\beta}_{3d}^{A_n(m)})$ is calculated from (13 f to i) associated with stresses. We have

$$u_{1d}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}\frac{s_{1d}^{(m)}}{\mu_m}\left(2K_0\left[s_{2d}^{(m)} - \frac{s_m s_{3d}^{(m)}}{(1+s_m)^2} + \frac{r_m s_{4d}^{(m)}}{(1+r_m)^2}\right] - \kappa_n^2\left[s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1+s_m} + \frac{s_{4d}^{(m)}}{1+r_m}\right] I_{1a} - \frac{s_m s_{3d}^{(m)}}{(1+s_m)^2}\frac{\partial \Pi_{(-s_m)}}{\partial |x_2|} + \frac{r_m s_{4d}^{(m)}}{(1+r_m)^2}\frac{\partial \Pi_{(-r_m)}}{\partial |x_2|}\right),$$

$$u_{1d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3}\frac{r_{1d}^{(m)}}{\mu_m} x_2\left((1+r_m)I_0 + r_m\kappa_n^2(r_m + \tilde{Q}_r)\Pi_{(-r_m)} - \kappa_n^2(1-\tilde{Q}_r)\Pi_1 \right);$$

$$u_{2d}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}\frac{s_{1d}^{(m)}}{\mu_m}(-1)^{m-1}\frac{\partial}{\partial x_1}\left(\left[s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1+s_m} + \frac{s_{4d}^{(m)}}{1+r_m}\right]\Pi_1 + \frac{s_{3d}^{(m)}}{1+s_m}\Pi_{(-s_m)} - \frac{s_{4d}^{(m)}}{1+r_m}\Pi_{(-r_m)}\right),$$

$$u_{2d}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3}\frac{r_{1d}^{(m)}}{\mu_m}\left\|\frac{\partial}{\partial x_1} + (-1)^{m-1}x_2\frac{\partial^2}{\partial x_1\partial|x_2|}\right\|\left((r_m+\tilde{Q}_r)\Pi_{(-r_m)} + (1-\tilde{Q}_r)\Pi_1\right);$$

$$u_{3d}^{A_n(m)A} = A_n\frac{s_{1d}^{(m)}}{\mu_m}\frac{\partial}{\partial x_1}\left(2K_0\left[\frac{s_{4d}^{(m)}}{(1+r_m)^2} - \frac{s_{3d}^{(m)}}{(1+s_m)^2}\right] + \kappa_n^2\left[s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1+s_m} + \frac{s_{4d}^{(m)}}{1+r_m}\right]I_{1a} - \frac{s_{3d}^{(m)}}{(1+s_m)^2}\frac{\partial\Pi_{(-s_m)}}{\partial|x_2|} + \frac{s_{4d}^{(m)}}{(1+r_m)^2}\frac{\partial\Pi_{(-r_m)}}{\partial|x_2|}\right),$$

$$u_{3d}^{A_n(m)B} = A_n\frac{r_{1d}^{(m)}}{\mu_m}\left\|x_2\kappa_n^2\frac{\partial}{\partial x_1} + (-1)^{m-1}4(1-\nu_m)\frac{\partial^2}{\partial x_1\partial|x_2|}\right\| \times\left((r_m+\tilde{Q}_r)\Pi_{(-r_m)} + (1-\tilde{Q}_r)\Pi_1\right);$$

$$\sigma_{11d}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3}2s_{1d}^{(m)}\frac{\partial}{\partial x_1}\left(2K_0\left[s_{2d}^{(m)} - \frac{s_m s_{3d}^{(m)}}{(1+s_m)^2} + \frac{r_m s_{4d}^{(m)}}{(1+r_m)^2}\right] - \kappa_n^2\left[s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1+s_m} + \frac{s_{4d}^{(m)}}{1+r_m}\right]I_{1a} - \frac{s_m s_{3d}^{(m)}}{(1+s_m)^2}\frac{\partial\Pi_{(-s_m)}}{\partial|x_2|} + \frac{r_m s_{4d}^{(m)}}{(1+r_m)^2}\frac{\partial\Pi_{(-r_m)}}{\partial|x_2|}\right),$$

$$\sigma_{11d}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3}2r_{1d}^{(m)}\frac{\partial}{\partial x_1}\left\{x_2\left[(1+r_m)I_0 + r_m\kappa_n^2(r_m+\tilde{Q}_r)\Pi_{(-r_m)} - \kappa_n^2(1-\tilde{Q}_r)\Pi_1\right] + (-1)^{m-1}2\nu_m\frac{\partial}{\partial|x_2|}\left((r_m+\tilde{Q}_r)\Pi_{(-r_m)} + (1-\tilde{Q}_r)\Pi_1\right)\right\};$$

$$\sigma_{22d}^{A_n(m)A} = -\frac{\partial A_n}{\partial x_3}2s_{1d}^{(m)}\frac{\partial}{\partial x_1}\left(2\left[s_{2d}^{(m)} - \frac{s_{3d}^{(m)}}{1+s_m} + \frac{s_{4d}^{(m)}}{1+r_m}\right]K_0 - \frac{s_{3d}^{(m)}}{1+s_m}\frac{\partial\Pi_{(-s_m)}}{\partial|x_2|} + \frac{s_{4d}^{(m)}}{1+r_m}\frac{\partial\Pi_{(-r_m)}}{\partial|x_2|}\right),$$

$$\sigma_{22d}^{A_n(m)B} = -\frac{\partial A_n}{\partial x_3}2r_{1d}^{(m)}\left\{x_2(1+r_m)\frac{\partial}{\partial x_1}\left[I_0 + \kappa_n^2(r_m+\tilde{Q}_r)\Pi_{(-r_m)}\right]\right.$$

$$+(-1)^{m-1}2(1-\nu_m)\frac{\partial^2}{\partial x_1 \partial |x_2|}\left((r_m+\tilde{Q}_r)\Pi_{(-r_m)}+(1-\tilde{Q}_r)\Pi_1\right)\Bigg\};$$

$$\sigma_{33d}^{A_n(m)A}=\frac{\partial A_n}{\partial x_3}2s_{1d}^{(m)}\frac{\partial}{\partial x_1}\Bigg(2K_0\left[\frac{s_{4d}^{(m)}}{(1+r_m)^2}-\frac{s_{3d}^{(m)}}{(1+s_m)^2}\right]$$
$$+\kappa_n^2\left[s_{2d}^{(m)}-\frac{s_{3d}^{(m)}}{1+s_m}+\frac{s_{4d}^{(m)}}{1+r_m}\right]I_{1a}-\frac{s_{3d}^{(m)}}{(1+s_m)^2}\frac{\partial \Pi_{(-s_m)}}{\partial |x_2|}+\frac{s_{4d}^{(m)}}{(1+r_m)^2}\frac{\partial \Pi_{(-r_m)}}{\partial |x_2|}\Bigg),$$

$$\sigma_{33d}^{A_n(m)B}=\frac{\partial A_n}{\partial x_3}2r_{1d}^{(m)}\left\| x_2\kappa_n^2\frac{\partial}{\partial x_1}+(-1)^{m-1}2(2-\nu_m)\frac{\partial^2}{\partial x_1\partial |x_2|}\right\|$$
$$\times\left((r_m+\tilde{Q}_r)\Pi_{(-r_m)}+(1-\tilde{Q}_r)\Pi_1\right);$$

$$\sigma_{12d}^{A_n(m)A}=\frac{\partial A_n}{\partial x_3}2s_{1d}^{(m)}(-1)^m\Bigg(s_{2d}^{(m)}I_0-\kappa_n^2\left[s_{2d}^{(m)}-\frac{s_{3d}^{(m)}}{1+s_m}+\frac{s_{4d}^{(m)}}{1+r_m}\right]\Pi_1$$
$$+\frac{s_m\kappa_n^2 s_{3d}^{(m)}}{1+s_m}\Pi_{(-s_m)}-\frac{r_m\kappa_n^2 s_{4d}^{(m)}}{1+r_m}\Pi_{(-r_m)}\Bigg),$$

$$\sigma_{12d}^{A_n(m)B}=\frac{\partial A_n}{\partial x_3}2r_{1d}^{(m)}\left\|1+x_2(-1)^{m-1}\frac{\partial}{\partial |x_2|}\right\|$$
$$\times\left((r_m+1)I_0+\kappa_n^2 r_m(r_m+\tilde{Q}_r)\Pi_{(-r_m)}-\kappa_n^2(1-\tilde{Q}_r)\Pi_1\right);$$

$$\sigma_{13d}^{A_n(m)A}=-A_n 2s_{1d}^{(m)}\kappa_n^2\Bigg(2K_0\left[s_{2d}^{(m)}-\frac{s_m s_{3d}^{(m)}}{(1+s_m)^2}+\frac{r_m s_{4d}^{(m)}}{(1+r_m)^2}\right]$$
$$-\kappa_n^2\left[s_{2d}^{(m)}-\frac{s_{3d}^{(m)}}{1+s_m}+\frac{s_{4d}^{(m)}}{1+r_m}\right]I_{1a}-\frac{s_m s_{3d}^{(m)}}{(1+s_m)^2}\frac{\partial \Pi_{(-s_m)}}{\partial |x_2|}+\frac{r_m s_{4d}^{(m)}}{(1+r_m)^2}\frac{\partial \Pi_{(-r_m)}}{\partial |x_2|}\Bigg),$$

$$\sigma_{13d}^{A_n(m)B}=-A_n 2r_{1d}^{(m)}\left\| x_2\kappa_n^2+2(1-\nu_m)(-1)^{m-1}\frac{\partial}{\partial |x_2|}\right\|$$
$$\times\left((r_m+1)I_0+\kappa_n^2 r_m(r_m+\tilde{Q}_r)\Pi_{(-r_m)}-\kappa_n^2(1-\tilde{Q}_r)\Pi_1\right);$$

$$\sigma_{23d}^{A_n(m)A}=A_n 2s_{1d}^{(m)}\kappa_n^2(-1)^m\frac{\partial}{\partial x_1}\left(\left[s_{2d}^{(m)}-\frac{s_{3d}^{(m)}}{1+s_m}+\frac{s_{4d}^{(m)}}{1+r_m}\right]\Pi_1\right.$$

$$+\frac{s_{3d}^{(m)}}{1+s_m}\Pi_{(-s_m)}-\frac{s_{4d}^{(m)}}{1+r_m}\Pi_{(-r_m)}\Bigg),$$

$$\sigma_{23d}^{A_n(m)B}=A_n 2r_{1d}^{(m)}\frac{\partial}{\partial x_1}\Big\{2(1-\nu_m)(1+r_m)\big[I_0+\kappa_n^2(r_m+\tilde{Q}_r)\Pi_{(-r_m)}\big]$$

$$+\kappa_n^2\left\|1+x_2(-1)^{m-1}\frac{\partial}{\partial|x_2|}\right\|\Big((r_m+\tilde{Q}_r)\Pi_{(-r_m)}+(1-\tilde{Q}_r)\Pi_1\Big)\Big\}; \qquad \text{(B.4)}$$

$$\Pi_{(-z)}\equiv\int_{-\infty}^{\infty}\frac{e^{-\sqrt{k_1^2+\kappa_n^2}|x_2|}}{k_1^2-z\kappa_n^2}e^{ik_1x_1}dk_1 .$$

The couple $(\overline{\alpha}_{3e}^{A_n(m)},\overline{\beta}_{3e}^{A_n(m)})$ is obtained from (13 *l*, *m*, *p* and *o*) associated with stresses. We have

$$u_{1e}^{A_n(m)A}=\frac{\partial A_n}{\partial x_3}\frac{s_{1e}^{(m)}}{\mu_m}\Big(2s_{2e}^{(m)}K_0+\kappa_n^2\big[s_{3e}^{(m)}-s_{2e}^{(m)}\big]I_{1a}\Big),$$

$$u_{1e}^{A_n(m)B}=\frac{\partial A_n}{\partial x_3}\frac{r_{1e}^{(m)}}{\mu_m}x_2\Big(I_0-2\kappa_n^2\Pi_1\Big);$$

$$u_{2e}^{A_n(m)A}=\frac{\partial A_n}{\partial x_3}\frac{s_{1e}^{(m)}}{\mu_m}(-1)^{m-1}\left(is_{3e}^{(m)}I_{1c}-\big[s_{3e}^{(m)}-s_{2e}^{(m)}\big]\frac{\partial\Pi_1}{\partial x_1}\right),$$

$$u_{2e}^{A_n(m)B}=\frac{\partial A_n}{\partial x_3}\frac{r_{1e}^{(m)}}{\mu_m}\left\|1+x_2(-1)^{m-1}\frac{\partial}{\partial|x_2|}\right\|\left(iI_{1c}-2\frac{\partial\Pi_1}{\partial x_1}\right);$$

$$u_{3e}^{A_n(m)A}=A_n\frac{s_{1e}^{(m)}}{\mu_m}\left(s_{3e}^{(m)}\frac{\partial}{\partial|x_2|}\left[\frac{\partial\Pi_1}{\partial x_1}-iI_{1c}\right]+\kappa_n^2\big[s_{2e}^{(m)}-s_{3e}^{(m)}\big]\frac{\partial I_{1a}}{\partial x_1}\right),$$

$$u_{3e}^{A_n(m)B}=A_n\frac{r_{1e}^{(m)}}{\mu_m}\left\|x_2\kappa_n^2+4(1-\nu_m)(-1)^{m-1}\frac{\partial}{\partial|x_2|}\right\|\left(2\frac{\partial\Pi_1}{\partial x_1}-iI_{1c}\right);$$

$$\sigma_{11e}^{A_n(m)A}=\frac{\partial A_n}{\partial x_3}2s_{1e}^{(m)}\frac{\partial}{\partial x_1}\Big(2s_{2e}^{(m)}K_0+\kappa_n^2\big[s_{3e}^{(m)}-s_{2e}^{(m)}\big]I_{1a}\Big),$$

$$\sigma_{11e}^{A_n(m)B}=\frac{\partial A_n}{\partial x_3}2r_{1e}^{(m)}\left(x_2\frac{\partial}{\partial x_1}\big[I_0-2\kappa_n^2\Pi_1\big]+(-1)^{m-1}2\nu_m\frac{\partial}{\partial|x_2|}\left[2\frac{\partial\Pi_1}{\partial x_1}-iI_{1c}\right]\right);$$

$$\sigma_{22e}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1e}^{(m)} \left(is_{3e}^{(m)} \frac{\partial I_{1c}}{\partial |x_2|} + 2\left[s_{3e}^{(m)} - s_{2e}^{(m)}\right] \frac{\partial K_0}{\partial x_1} \right),$$

$$\sigma_{22e}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1e}^{(m)} \left(x_2 \left[\frac{\partial I_0}{\partial x_1} + i\kappa_n I_{1c} \right] + (-1)^m 2(1-\nu_m) \frac{\partial}{\partial |x_2|} \left[2\frac{\partial \Pi_1}{\partial x_1} - iI_{1c} \right] \right);$$

$$\sigma_{33e}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1e}^{(m)} \left(s_{3e}^{(m)} \frac{\partial}{\partial |x_2|} \left[\frac{\partial \Pi_1}{\partial x_1} - iI_{1c} \right] + \kappa_n^2 \left[s_{2e}^{(m)} - s_{3e}^{(m)}\right] \frac{\partial I_{1a}}{\partial x_1} \right),$$

$$\sigma_{33e}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1e}^{(m)} \left\| x_2\kappa_n^2 + 2(2-\nu_m)(-1)^{m-1} \frac{\partial}{\partial |x_2|} \right\| \left(2\frac{\partial \Pi_1}{\partial x_1} - iI_{1c} \right);$$

$$\sigma_{12e}^{A_n(m)A} = \frac{\partial A_n}{\partial x_3} 2s_{1e}^{(m)} (-1)^m \left(s_{2e}^{(m)} I_0 + \kappa_n^2 \left[s_{3e}^{(m)} - s_{2e}^{(m)}\right] \Pi_1 \right),$$

$$\sigma_{12e}^{A_n(m)B} = \frac{\partial A_n}{\partial x_3} 2r_{1e}^{(m)} \left\| 1 + x_2(-1)^{m-1} \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - 2\kappa_n^2 \Pi_1 \right);$$

$$\sigma_{13e}^{A_n(m)A} = -A_n 2s_{1e}^{(m)} \kappa_n^2 \left(2s_{2e}^{(m)} K_0 + \kappa_n^2 \left[s_{3e}^{(m)} - s_{2e}^{(m)}\right] I_{1a} \right),$$

$$\sigma_{13e}^{A_n(m)B} = -A_n 2r_{1e}^{(m)} \left\| x_2\kappa_n^2 + 2(1-\nu_m)(-1)^{m-1} \frac{\partial}{\partial |x_2|} \right\| \left(I_0 - 2\kappa_n^2 \Pi_1 \right);$$

$$\sigma_{23e}^{A_n(m)A} = A_n 2s_{1e}^{(m)} \kappa_n^2 (-1)^m \left(is_{3e}^{(m)} I_{1c} + \left[s_{2e}^{(m)} - s_{3e}^{(m)}\right] \frac{\partial \Pi_1}{\partial x_1} \right),$$

$$\sigma_{23e}^{A_n(m)B} = A_n 2r_{1e}^{(m)} \left(\frac{\partial}{\partial x_1} \left[2(1-\nu_m) I_0 + 2\kappa_n^2 \Pi_1 \right] - i\kappa_n^2 (3-2\nu_m) I_{1c} \right.$$

$$\left. + x_2\kappa_n^2 (-1)^{m-1} \frac{\partial}{\partial |x_2|} \left[2\frac{\partial \Pi_1}{\partial x_1} - iI_{1c} \right] \right). \quad \text{(B.5)}$$