

**A STUDY OF THE ELASTIC FIELDS OF INTERFACIAL EDGE  
DISLOCATIONS (STRAIGHT AND SINUSOIDAL) USING  
GALERKIN VECTORS WITH THREE-DIMENSIONAL  
BIHARMONIC FUNCTIONS IN FOURIER FORMS**

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**ABSTRACT**

In this study, we consider two elastic solids ( $S_1$ ) and ( $S_2$ ), of infinite sizes, welded along a non-planar surface  $S$  in the form of a corrugated sheet; more specifically, with respect to a Cartesian coordinate system  $x_i$ , the interface has the same sinusoidal shape  $\xi = \xi_n \sin \kappa_n x_3$  in the  $x_2 x_3$  - planes and is rectilinear in the  $x_1 x_2$  - planes. We investigate the elastic fields (displacement and stress) due to a dislocation lying on that interface at the origin and running indefinitely along the  $x_3$  - direction. The approach used is to treat the elastic fields as the difference of two quantities : 1) the first corresponds to the elastic fields of a sinusoidal dislocation at the origin in an infinitely extended homogeneous medium and 2) the second satisfies the equilibrium equations with a discontinuity, when crossing the interface, identical to that given by the elastic fields of the sinusoidal dislocation from the change in the elastic constants on the passage from ( $S_2$ ) to ( $S_1$ ).

This second quantity is set using Galerkin vectors whose components are expressed in the form of Fourier series and integrals. Then equations are written that reflect the continuity of the elastic fields at the crossing of the interface. These interface boundary conditions split into two distinct groups: those corresponding to a planar interface with a straight edge dislocation at the origin and those (in the linear approximation with respect to  $\xi$ , assuming  $\xi$  to be small) proportional to the sinusoid or its spatial derivative with respect to  $x_3$ . We then restrict our treatment by satisfying only to the boundary conditions associated with a planar interface with a straight edge dislocation.

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The displacement and stress fields of an interface straight edge dislocation, thus obtained, reflect the presence of the Dirac delta function in the shear stresses on the interface. Finally, a comparison is made of our findings with those previously published on the same subject.

**Keywords :** *linear elasticity, interface dislocations, Galerkin vector, three-dimensional biharmonic functions, Fourier forms, linear systems of equations*

## RÉSUMÉ

**Une étude des champs élastiques de dislocations coins droites et sinusoidales utilisant des vecteurs de Galerkin avec des composantes tridimensionnelles biharmoniques dans la forme de Fourier**

Dans la présente étude, on considère deux solides élastiques ( $S1$ ) et ( $S2$ ), de tailles infinies, soudés suivant une surface non plane ayant la forme d'une tôle ondulée; plus précisément, par rapport à un système de coordonnées cartésien  $x_i$ , l'interface a une forme sinusoidale identique  $\xi = \xi_n \sin \kappa_n x_3$  dans les plans  $x_2 x_3$  et rectiligne dans les plans  $x_1 x_2$ . On étudie les champs élastiques (déplacement et contrainte) d'une dislocation couchée sur cette interface à l'origine et courant indéfiniment dans la direction  $x_3$ . La démarche utilisée est de considérer les champs élastiques comme la différence de deux grandeurs : 1) la première correspond aux champs élastiques d'une dislocation sinusoidale dans un milieu homogène infiniment étendu et 2) la seconde satisfait aux équations d'équilibre avec une discontinuité, à la traversée de l'interface, identique à celle mesurée dans les expressions des champs élastiques de la dislocation sinusoidale et qui résulte du changement des constantes élastiques au passage de l'interface, du solide ( $S2$ ) vers le solide ( $S1$ ).

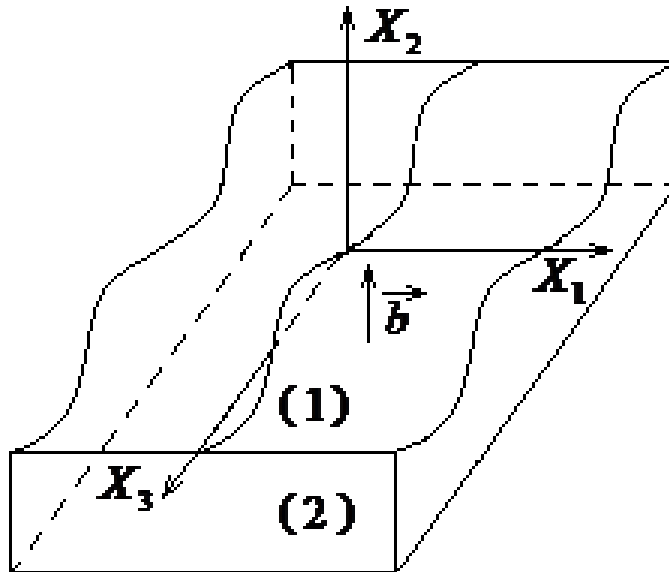
Cette deuxième quantité est définie à l'aide de vecteurs de Galerkin dont les composantes sont développées dans la forme de Fourier. On pose ensuite des équations traduisant la continuité des champs élastiques à la traversée de l'interface. Ces conditions aux bords pour l'interface se répartissent en deux groupes distincts: 1) celles qui correspondent à une interface plane avec une dislocation coin droite à l'origine et 2) celles qui (dans l'approximation linéaire par rapport à  $\xi$ , supposé petit) sont proportionnelles à la sinusoidale ou à sa dérivée spatiale par rapport à  $x_3$ . Nous restreignons alors notre traitement en satisfaisant uniquement les conditions aux bords associées à une interface plane avec une dislocation coin droite.

Les champs élastiques d'une dislocation d'interface coin droite, ainsi obtenus, rendent compte de la présence de la fonction delta de Dirac dans les contraintes de cisaillement sur l'interface. Enfin, une comparaison est faite de nos résultats avec ceux publiés antérieurement sur le même sujet.

**Mots-clés :** *élasticité linéaire, dislocations d'interface, Vecteur de Galerkin, fonctions biharmoniques à trois dimensions, expansions en séries de Fourier, systèmes d'équations linéaires*

## I - INTRODUCTION

Consider a pair of different solids  $S_1$  and  $S_2$ , of infinite sizes, welded along a non-planar sinusoidal surface  $S$  defined by the running point  $P_S(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$ , in such a way that  $S_1$  and  $S_2$  occupy the regions  $x_2 > \xi_n \sin \kappa_n x_3$  and  $x_2 < \xi_n \sin \kappa_n x_3$ , respectively. The situation is shown in the **Figure 1** where  $S_1$  and  $S_2$  are confined for illustration purpose in a parallelepiped of finite sizes.



**Figure 1 :** *Two elastic mediums (1) and (2) welded along a non-planar sinusoidal surface and containing an interface sinusoidal dislocation at the origin. The dislocation lies in the  $Ox_2x_3$  - plane and runs indefinitely in the  $x_3$  - direction.*

The present study aims at providing expressions for the displacement and stress fields of sinusoidal dislocations, lying on that interface at the origin, extending indefinitely in the  $x_3$  - direction and spreading in the  $Ox_2x_3$  - plane in the sinusoidal form  $A_n = \xi_n \sin \kappa_n x_3$ . The dislocation is edge on average for Burgers vectors in the  $x_1$  and  $x_2$  directions and screw for a Burgers vector in the  $x_3$  - direction. In the present report, we restrict ourselves to a Burgers vector  $\vec{b} = (0, b, 0)$  in the  $x_2$  - direction. Using the results of such a study is at several levels: 1) the elastic fields due to an arbitrary form of interface dislocation in  $x_2x_3$  planes with the same Burgers vector can be derived by superposition (Fourier series expansion ); 2) a non-planar large interface crack loaded in tension in the  $x_2$  - direction and propagating in the  $x_1$  - direction may be represented mathematically by a continuous array of long non-straight dislocations with infinitesimal Burgers vectors  $\vec{b} = (0, b, 0)$ . Denote by  $\vec{u}^{(m)}$  and  $(\sigma)^{(m)}$  ( $m = 1$  and  $2$ ) the displacement and stress fields in the solid ( $m$ ) due to the interface sinusoidal dislocation. We assume that the following description applies :

- $\vec{u}^{(m)}$  and  $(\sigma)^{(m)}$  are continuous at the crossing of the interface

$$\vec{u}^{(1)}(P_S) = \vec{u}^{(2)}(P_S) \text{ and } (\sigma)^{(1)}(P_S) = (\sigma)^{(2)}(P_S) \quad (1)$$

- Far from the dislocation and the interface, the elastic fields in the medium ( $m$ ) correspond to those of a sinusoidal dislocation in an infinitely extended homogeneous solid with the equal elastic constants, that we denote by  $\vec{u}^{(m)\infty}$  and  $(\sigma)^{(m)\infty}$ , hence

$$\begin{aligned} \vec{u}^{(m)} &\rightarrow \vec{u}^{(m)\infty} \\ (\sigma)^{(m)} &\rightarrow (\sigma)^{(m)\infty} \end{aligned} \quad (2)$$

when one moves far away in the  $x_2$  - direction.

- The elastic fields may be expressed in the form

$$\begin{aligned} \vec{u}^{(m)} &= \vec{u}^{(m)\infty} - \vec{u}^{(m)W} \\ (\sigma)^{(m)} &= (\sigma)^{(m)\infty} - (\sigma)^{(m)W} \end{aligned} \quad (3)$$

where  $\vec{u}^{(m)W}$  and  $(\sigma)^{(m)W}$  satisfy the equations of equilibrium and posses the properties that follow.

$$\Delta \bar{u}^\infty (P_S) \equiv \bar{u}^{(2)\infty} - \bar{u}^{(1)\infty} = \bar{u}^{(2)W} - \bar{u}^{(1)W} \equiv \Delta \bar{u}^W (P_S)$$

$$(\Delta \sigma)^\infty (P_S) \equiv (\sigma)^{(2)\infty} - (\sigma)^{(1)\infty} = (\sigma)^{(2)W} - (\sigma)^{(1)W} \equiv (\Delta \sigma)^W (P_S), \quad (4)$$

these conditions ensure the continuity of the elastic fields across the interface.

- $\bar{u}^{(m)W}$  and  $(\sigma)^{(m)W}$  cancel far from the dislocation and interface; this means that

$$\begin{aligned} \bar{u}^{(m)W} &\rightarrow 0 \\ (\sigma)^{(m)W} &\rightarrow 0 \end{aligned} \quad (5)$$

when  $|x_2| \rightarrow \infty$ ; this ensures the veracity of condition (2) above.

The elastic fields  $\bar{u}^{(m)}$  and  $(\sigma)^{(m)}$  thus obtained are expected to be valuable representations of the physical situation illustrated in the **Figure 1**. The associated  $\bar{u}^{(m)W}$  and  $(\sigma)^{(m)W}$  are investigated with the help of Galerkin vectors and corresponding equations of equilibrium. The methodology in Section 2 is essentially as follows: in Section 2.1,  $\Delta \bar{u}^\infty (P_S)$  and  $(\Delta \sigma)^\infty (P_S)$  are expressed in a Fourier series form that involves terms with  $\exp(i\vec{k} \cdot \vec{x})$ , with  $\vec{k} = (k_1, k_2, k_3)$  ( $k_i$  real numbers) and  $\vec{x} = (x_1, x_2, x_3)$  vector position; in Section 2.2, a Galerkin vector with components involving  $\exp(i\vec{k} \cdot \vec{x})$  is considered. The associated elastic fields also consist of terms proportional to  $\exp(i\vec{k} \cdot \vec{x})$ . These are managed in the equal Fourier series form. Then equations of the type (4) can be posed. In Section 3, the search for the appropriate elastic fields  $\bar{u}^{(m)W}$  and  $(\sigma)^{(m)W}$  is initiated restricting ourselves, in the present paper, to the interface straight edge dislocation. A complete study requires additional works; this will be the subject of a separate paper. In Section 4, the calculation results are discussed and confronted to previous studies. A conclusion is made in Section 5.

## II - METHODOLOGY

### II-1. Interface boundary values carried by the elastic fields of a sinusoidal dislocation in an homogeneous solid

The elastic fields due to a sinusoidal edge dislocation ( $\vec{b} = (0, b, 0)$ ) lying in the  $Ox_2x_3$  - plane in the sinusoidal form  $A_n(x_3) = \xi_n \sin \kappa_n x_3$  have been provided in infinite series forms by [1].

In a similar way as in our previous studies [2 - 6], we shall assume  $\xi_n$  small and limit the elastic solutions up to terms of first order with respect to  $\xi_n$ . In this way, the elastic fields consist of two terms:

$$\begin{aligned}\bar{u}^{(m)\infty} &= \bar{u}^{(0)(m)\infty} + \bar{u}^{A_n(m)\infty} \\ (\sigma)^{(m)\infty} &= (\sigma)^{(0)(m)\infty} + (\sigma)^{A_n(m)\infty}\end{aligned}\quad (6)$$

where  $\bar{u}^{(0)(m)\infty}$  and  $(\sigma)^{(0)(m)\infty}$  are of zero order with respect to  $\xi_n$  corresponding to the fields of a straight edge dislocation;  $\bar{u}^{A_n(m)\infty}$  and  $(\sigma)^{A_n(m)\infty}$  are proportional to the sinusoid  $A_n(x_3)$  or to its spatial derivative  $\partial A_n / \partial x_3$ .

Our purpose here is to write down the differences  $\Delta \bar{u}^\infty$  and  $(\Delta \sigma)^\infty$  (4) on crossing the interface at arbitrary point  $P_s(x_1, x_2 = \xi_n \sin \kappa_n x_3, x_3)$ . We use the notation  $x_2 = \xi$  ( $\xi$  small) and take the MacLaurin series expansions of the elastic fields up to terms of first order with respect to  $\xi$ ; this means that

$$\begin{aligned}\Delta \bar{u}^\infty(x_1, x_2 = \xi, x_3) &= \Delta \bar{u}^\infty(x_1, 0, x_3) + \frac{\partial \Delta \bar{u}^\infty}{\partial x_2}(x_1, 0, x_3)\xi, \\ (\Delta \sigma)^\infty(x_1, x_2 = \xi, x_3) &= (\Delta \sigma)^\infty(x_1, 0, x_3) + \frac{\partial (\Delta \sigma)^\infty}{\partial x_2}(x_1, 0, x_3)\xi.\end{aligned}\quad (7)$$

$\bar{u}^{(m)\infty}$  and  $(\sigma)^{(m)\infty}$  are taken from our previous works [1, 4,5]; we obtain ( $u_i$  is the  $i$ -component of vector  $\bar{u}$  and  $\sigma_{ij}$  the  $ij$ -element of the stress matrix  $(\sigma)$ ;  $i, j= 1$  to 3)

$$\begin{aligned}\Delta u_i^\infty(x_1, x_2 = \xi, x_3) &= \Delta u_i^{(0)\infty} + \Delta u_i^{A_n\infty} \\ \Delta \sigma_{ij}^\infty(x_1, x_2 = \xi, x_3) &= \Delta \sigma_{ij}^{(0)\infty} + \Delta \sigma_{ij}^{A_n\infty}\end{aligned}$$

as

$$\begin{aligned}\Delta u_1^{(0)\infty} &= \frac{bC_\nu}{4\pi} \ln|x_1| = -\frac{bC_\nu}{8\pi} \int_{-\infty}^{\infty} \frac{1}{|k_1|} e^{ik_1 x_1} dk_1 \\ \Delta u_1^{A_n\infty} &= \frac{bC_\nu \kappa_n A_n}{4\pi|x_1|} (\kappa_n|x_1|K_0 + K_1)\xi = -\frac{bC_\nu A_n}{8\pi} \int_{-\infty}^{\infty} \frac{k_1^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \xi\end{aligned}$$

$$\Delta u_2^{(0)\infty} = \frac{bC_v}{4\pi x_1} \xi = -\frac{ibC_v}{8\pi} \int_{-\infty}^{\infty} \operatorname{sgn}(k_1) e^{ik_1 x_1} dk_1 \xi$$

$$\Delta u_2^{A_n\infty} = -\frac{bC_v \kappa_n A_n}{4\pi} \operatorname{sgn}(x_1) K_1 = \frac{ibC_v A_n}{8\pi} \int_{-\infty}^{\infty} \frac{k_1}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1$$

$$\Delta u_3^{(0)\infty} = 0$$

$$\Delta u_3^{A_n\infty} = -\frac{bC_v \kappa_n}{4\pi} \frac{\partial A_n}{\partial x_3} \operatorname{sgn}(x_1) K_1 \xi = \frac{ibC_v}{8\pi} \frac{\partial A_n}{\partial x_3} \int_{-\infty}^{\infty} \frac{k_1}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \xi$$

$$\Delta \sigma_{11}^{(0)\infty} = \frac{C_2 - C_1}{x_1} = -2Q_b \int_{-\infty}^{\infty} \operatorname{sgn}(k_1) e^{ik_1 x_1} dk_1$$

$$\begin{aligned} \Delta \sigma_{11}^{A_n\infty} &= \frac{(C_2 - C_1) \kappa_n A_n}{x_1} \left( 3\kappa_n K_0 + \frac{(6 + \kappa_n^2 x_1^2) K_1}{|x_1|} \right) \xi \\ &= 2Q_b A_n \int_{-\infty}^{\infty} \frac{k_1 (3k_1^2 + 2\kappa_n^2)}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \xi \end{aligned}$$

$$\Delta \sigma_{22}^{(0)\infty} = \frac{C_2 - C_1}{x_1} = -2Q_b \int_{-\infty}^{\infty} \operatorname{sgn}(k_1) e^{ik_1 x_1} dk_1$$

$$\begin{aligned} \Delta \sigma_{22}^{A_n\infty} &= -\frac{(C_2 - C_1) \kappa_n A_n}{x_1} \left( \kappa_n K_0 + \frac{2K_1}{|x_1|} \right) \xi \\ &= -2Q_b A_n \int_{-\infty}^{\infty} k_1 \sqrt{k_1^2 + \kappa_n^2} e^{ik_1 x_1} dk_1 \xi \end{aligned}$$

$$\Delta \sigma_{33}^{(0)\infty} = -\frac{8iQ_c}{x_1} = -4Q_c \int_{-\infty}^{\infty} \operatorname{sgn}(k_1) e^{ik_1 x_1} dk_1$$

$$\begin{aligned} \Delta \sigma_{33}^{A_n\infty} &= -\frac{\kappa_n A_n}{x_1} \left( 8iQ_c \kappa_n K_0 + \left( \frac{16iQ_c}{|x_1|} + \kappa_n^2 (C_2 - C_1) |x_1| \right) K_1 \right) \xi \\ &= 2A_n \int_{-\infty}^{\infty} \frac{k_1 (2Q_c k_1^2 + (2Q_c + Q_b) \kappa_n^2)}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \xi \end{aligned}$$

$$\Delta \sigma_{12}^{(0)\infty} = \frac{C_2 - C_1}{x_1^2} \xi = 2iQ_b \int_{-\infty}^{\infty} |k_1| e^{ik_1 x_1} dk_1 \xi$$

$$\begin{aligned}
\Delta \sigma_{12}^{A_n \infty} &= \kappa_n A_n \left( 4iQ_c \kappa_n K_0 - \frac{(C_2 - C_1)K_1}{|x_1|} \right) \\
&= -2iA_n \int_{-\infty}^{\infty} \frac{Q_b k_1^2 + (Q_b - Q_c)\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \\
\Delta \sigma_{13}^{(0)\infty} &= 0 \\
\Delta \sigma_{13}^{A_n \infty} &= 4i\kappa_n \frac{\partial A_n}{\partial x_3} \left( Q_b \kappa_n K_0 + \frac{(2Q_b - Q_c)K_1}{|x_1|} \right) \xi \\
&= 2i \frac{\partial A_n}{\partial x_3} \int_{-\infty}^{\infty} \frac{(Q_c - 2Q_b)k_1^2 + (Q_c - Q_b)\kappa_n^2}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \xi \\
\Delta \sigma_{23}^{(0)\infty} &= 0 \\
\Delta \sigma_{23}^{A_n \infty} &= -4iQ_c \kappa_n \frac{\partial A_n}{\partial x_3} \operatorname{sgn}(x_1) K_1 = -2Q_c \frac{\partial A_n}{\partial x_3} \int_{-\infty}^{\infty} \frac{k_1}{\sqrt{k_1^2 + \kappa_n^2}} e^{ik_1 x_1} dk_1 \quad (8)
\end{aligned}$$

where

$$\begin{aligned}
C_v &= [1/(1 - \nu_1) - 1/(1 - \nu_2)], \quad Q_b = i(C_2 - C_1)/4, \\
Q_c &= i(\nu_2 C_2 - \nu_1 C_1)/4, \quad C_m = b\mu_m / 2\pi(1 - \nu_m);
\end{aligned}$$

$K_i$  is the  $i$ th-order modified Bessel function with argument  $\kappa_n |x_1|$  and  $\operatorname{sgn}(k_1) = k_1/|k_1|$ ;  $\mu_m$  and  $\nu_m$  are shear modulus and Poisson's ratio. In the various expressions in (8), constant terms are omitted.

## II-2. Galerkin vectors and interface boundary conditions

A Galerkin vector  $\vec{v}(\vec{x})$  is a vector whose components are biharmonic spatial functions ( $\Delta \Delta \vec{v} = 0$ ;  $\Delta$  Laplace operator) in order to satisfy the equilibrium equations with zero body forces. Then the associated displacement  $\vec{u}$  is expressed as

$$2\mu\vec{u} = 2(1 - \nu)\Delta\vec{V} - \vec{\nabla}(\vec{\nabla} \cdot \vec{V}) \quad (9)$$

where  $\mu$  and  $\nu$  are shear modulus and Poisson's ratio respectively;  $\vec{\nabla}$  is the operator nabla,  $\vec{\nabla} = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ .



The stress field  $(\sigma)$  is obtained from the displacement  $\vec{u}$  by partial differentiation with respect to coordinates  $x_i$ . The matter is treated in a number of books (see [7-11], among many others). For the present problem, we arrive at Galerkin vectors with only one non-zero  $x_2$  – component, arranged in the form

$$V_2(\vec{x}) = \bar{\alpha}_2(\vec{k})e^{i\vec{k}\cdot\vec{x}} + \bar{\beta}_2(\vec{k})x_2e^{i\vec{k}\cdot\vec{x}} \tag{10}$$

under the condition  $\vec{k}^2 = k_1^2 + k_2^2 + k_3^2 = 0$  that ensures the biharmonicity of  $V_2$ . For  $V_2$  to cancel far from the interface, we write

$$k_2 = k_2^{(m)} \equiv (-1)^{m-1}i\sqrt{k_1^2 + k_3^2} \tag{11}$$

with  $m = 1$  when  $x_2 > \xi_n \sin \kappa_n x_3$  (half-space 1) and  $m = 2$  when  $x_2 < \xi_n \sin \kappa_n x_3$  (half-space 2). We use the notations

$$\vec{k}^{(m)} \equiv (k_1, k_2^{(m)}, k_3), \bar{\alpha}_2^{(m)} \equiv \bar{\alpha}_2(\vec{k}^{(m)}), \bar{\beta}_2^{(m)} \equiv \bar{\beta}_2(\vec{k}^{(m)});$$

hence for half-space 1 ( $x_2 > \xi_n \sin \kappa_n x_3$ ), solid (1)

$$V_2(\vec{x}) \equiv V_2^{(1)}(\vec{x}) = \bar{\alpha}_2^{(1)}e^{i\vec{k}^{(1)}\cdot\vec{x}} + \bar{\beta}_2^{(1)}x_2e^{i\vec{k}^{(1)}\cdot\vec{x}}$$

and for half-space 2 ( $x_2 < \xi_n \sin \kappa_n x_3$ ), solid (2)

$$V_2(\vec{x}) \equiv V_2^{(2)}(\vec{x}) = \bar{\alpha}_2^{(2)}e^{i\vec{k}^{(2)}\cdot\vec{x}} + \bar{\beta}_2^{(2)}x_2e^{i\vec{k}^{(2)}\cdot\vec{x}}.$$

The elastic fields corresponding to  $V_2$  (10) may be first calculated starting with (9); then, more general forms  $\vec{u}^{(m)V}$  and  $(\sigma)^{(m)V}$  are constructed from the previous ones by superposition over  $k_1$  and  $k_3$  (here the superscript  $V$  is just a notation, not to be confused with  $\|\vec{V}\|$ ). For  $\vec{u}^{(m)V}$  and  $(\sigma)^{(m)V}$  to conform with  $\vec{u}^{(m)\infty}$  and  $(\sigma)^{(m)\infty}$  (6), the summation over  $k_1$  is continuous and that over  $k_3$  is discrete.  $k_3$  takes three values:  $-\kappa_n, 0, \kappa_n$ . The fields corresponding to  $k_3 = 0$  are denoted  $\vec{u}^{(0)(m)V}$  and  $(\sigma)^{(0)(m)V}$  and terms associated with  $k_3 = -\kappa_n$  and  $\kappa_n$  are merged to form expressions denoted  $\vec{u}^{A_n(m)V}$  and  $(\sigma)^{A_n(m)V}$ ; this is made possible by requiring that

$$\bar{\alpha}_2^{(m)}(\kappa_n) \equiv -\bar{\alpha}_2^{(m)}(-\kappa_n), \quad \bar{\beta}_2^{(m)}(\kappa_n) \equiv -\bar{\beta}_2^{(m)}(-\kappa_n). \quad (12)$$

In (12),  $\bar{\alpha}_2^{(m)}(\kappa_n)$  stands for  $\bar{\alpha}_2(k_1, k_2^{(m)}, \kappa_n) \cdot \bar{u}^{(0)(m)V}$  and  $(\sigma)^{(0)(m)V}$  are  $x_3$  - independent;  $\bar{u}^{A_n(m)V}$  and  $(\sigma)^{A_n(m)V}$  are proportional to the sinusoid  $A_n(x_3)$  or to its spatial derivative  $\partial A_n / \partial x_3$ . Here also, for points  $P_s$  on the interface,  $\Delta \bar{u}^V$  and  $(\Delta \sigma)^V$  are expanded up to terms of first order with respect to  $x_2 = \xi$  in a similar manner as in (8) for  $\Delta \bar{u}^\infty$  and  $(\Delta \sigma)^\infty$ . Requiring  $\Delta \bar{u}^V = \Delta \bar{u}^\infty$  and  $(\Delta \sigma)^V = (\Delta \sigma)^\infty$  lead to the following equations, writing first the conditions corresponding to  $k_3 = 0$  (i.e.  $\Delta u_i^{(0)V} = \Delta u_i^{(0)\infty}$  and  $\Delta \sigma_{ij}^{(0)V} = \Delta \sigma_{ij}^{(0)\infty}$ ).

$$\Delta u_1^{(0)V} = \Delta u_1^{(0)\infty} \Rightarrow$$

$$|k_1 \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} + \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) + \left( \frac{\bar{\beta}_2^{(2)}}{\mu_2} - \frac{\bar{\beta}_2^{(1)}}{\mu_1} \right) = -\frac{ibC_v \operatorname{sgn}(k_1)}{4\pi k_1^2} \quad (a)$$

$$|k_1 \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} - \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) + 2 \left( \frac{\bar{\beta}_2^{(2)}}{\mu_2} + \frac{\bar{\beta}_2^{(1)}}{\mu_1} \right) = 0 \quad (b)$$

$$\Delta u_2^{(0)V} = \Delta u_2^{(0)\infty} \Rightarrow$$

$$|k_1 \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} - \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) - 2 \left( \frac{(1-2\nu_2)\bar{\beta}_2^{(2)}}{\mu_2} + \frac{(1-2\nu_1)\bar{\beta}_2^{(1)}}{\mu_1} \right) = 0 \quad (c)$$

$$|k_1 \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} + \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) - \left( \frac{(1-4\nu_2)\bar{\beta}_2^{(2)}}{\mu_2} - \frac{(1-4\nu_1)\bar{\beta}_2^{(1)}}{\mu_1} \right) = \frac{ibC_v \operatorname{sgn}(k_1)}{4\pi k_1^2} \quad (d)$$

$$\Delta \sigma_{11}^{(0)V} = \Delta \sigma_{11}^{(0)\infty} \Rightarrow$$

$$|k_1 \left( \bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)} \right) + (1+2\nu_2)\bar{\beta}_2^{(2)} - (1+2\nu_1)\bar{\beta}_2^{(1)} = -2Q_b \frac{\operatorname{sgn}(k_1)}{k_1^2} \quad (e)$$

$$|k_1 \left( \bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)} \right) + 2(1+\nu_2)\bar{\beta}_2^{(2)} + 2(1+\nu_1)\bar{\beta}_2^{(1)} = 0 \quad (f)$$

$$\Delta \sigma_{22}^{(0)V} = \Delta \sigma_{22}^{(0)\infty} \Rightarrow$$

$$|k_1 \left( \bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)} \right) - (1-2\nu_2)\bar{\beta}_2^{(2)} + (1-2\nu_1)\bar{\beta}_2^{(1)} = 2Q_b \frac{\operatorname{sgn}(k_1)}{k_1^2} \quad (g)$$

$$|k_1 \left( \bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)} \right) + 2\nu_2\bar{\beta}_2^{(2)} + 2\nu_1\bar{\beta}_2^{(1)} = 0 \quad (h)$$

$$\Delta \sigma_{33}^{(0)V} = \Delta \sigma_{33}^{(0)\infty} \Rightarrow$$

$$v_2 \bar{\beta}_2^{(2)} - v_1 \bar{\beta}_2^{(1)} = -2Q_c \frac{\text{sgn}(k_1)}{k_1^2} \tag{i}$$

$$v_2 \bar{\beta}_2^{(2)} + v_1 \bar{\beta}_2^{(1)} = 0 \tag{j}$$

$$\Delta \sigma_{12}^{(0)V} = \Delta \sigma_{12}^{(0)\infty} \Rightarrow \text{(h) and (e) above} \tag{13}$$

In *Equations* (13 a to j) above,  $\bar{\alpha}_2^{(m)}$  stands for  $\bar{\alpha}_2(k_1, k_2^{(m)}, k_3 = 0)$ .

The conditions corresponding to  $\Delta u_i^{A_n V} = \Delta u_i^{A_n \infty}$  and  $\Delta \sigma_{ij}^{A_n V} = \Delta \sigma_{ij}^{A_n \infty}$  are now listed as :

$$\Delta u_1^{A_n V} = \Delta u_1^{A_n \infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2} \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} + \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) + \frac{\bar{\beta}_2^{(2)}}{\mu_2} - \frac{\bar{\beta}_2^{(1)}}{\mu_1} = 0 \tag{a}$$

$$\sqrt{k_1^2 + \kappa_n^2} \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} - \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) + 2 \left( \frac{\bar{\beta}_2^{(2)}}{\mu_2} + \frac{\bar{\beta}_2^{(1)}}{\mu_1} \right) = -\frac{bC_v \xi_n}{8\pi} \frac{k_1}{k_1^2 + \kappa_n^2} \tag{b}$$

$$\Delta u_2^{A_n V} = \Delta u_2^{A_n \infty} \Rightarrow \text{(c) and (d) (displayed below)}$$

$$\sqrt{k_1^2 + \kappa_n^2} \left( -\frac{\bar{\alpha}_2^{(2)}}{\mu_2} + \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) + 2 \left( \frac{(1 - 2v_2) \bar{\beta}_2^{(2)}}{\mu_2} + \frac{(1 - 2v_1) \bar{\beta}_2^{(1)}}{\mu_1} \right) = \frac{bC_v \xi_n}{8\pi} \frac{k_1}{k_1^2 + \kappa_n^2}$$

$$\sqrt{k_1^2 + \kappa_n^2} \left( \frac{\bar{\alpha}_2^{(2)}}{\mu_2} + \frac{\bar{\alpha}_2^{(1)}}{\mu_1} \right) - \frac{(1 - 4v_2) \bar{\beta}_2^{(2)}}{\mu_2} + \frac{(1 - 4v_1) \bar{\beta}_2^{(1)}}{\mu_1} = 0$$

$$\Delta u_3^{A_n V} = \Delta u_3^{A_n \infty} \Rightarrow \text{(a) and (b) above}$$

$$\Delta \sigma_{11}^{A_n V} = \Delta \sigma_{11}^{A_n \infty} \Rightarrow$$

$$k_1^2 \sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)}) + [(1 + 2v_2)k_1^2 + 2v_2 \kappa_n^2] \bar{\beta}_2^{(2)}$$

$$- [(1 + 2v_1)k_1^2 + 2v_1 \kappa_n^2] \bar{\beta}_2^{(1)} = 0 \tag{e}$$

$$k_1^2 \sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)}) + 2[(1 + v_2)k_1^2 + v_2 \kappa_n^2] \bar{\beta}_2^{(2)}$$

$$+ 2[(1 + v_1)k_1^2 + v_1 \kappa_n^2] \bar{\beta}_2^{(1)} = -\frac{iQ_b \xi_n k_1 (3k_1^2 + 2\kappa_n^2)}{k_1^2 + \kappa_n^2} \tag{f}$$

$$\Delta \sigma_{22}^{A_n V} = \Delta \sigma_{22}^{A_n \infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)}) - (1 - 2\nu_2) \bar{\beta}_2^{(2)} + (1 - 2\nu_1) \bar{\beta}_2^{(1)} = 0 \quad (g)$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)}) + 2(\nu_2 \bar{\beta}_2^{(2)} + \nu_1 \bar{\beta}_2^{(1)}) = -\frac{iQ_b \xi_n k_1}{k_1^2 + \kappa_n^2} \quad (h)$$

$$\Delta \sigma_{33}^{A_n V} = \Delta \sigma_{33}^{A_n \infty} \Rightarrow$$

$$\kappa_n^2 \sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)}) + [(1 + 2\nu_2) \kappa_n^2 + 2\nu_2 k_1^2] \bar{\beta}_2^{(2)}$$

$$- [(1 + 2\nu_1) \kappa_n^2 + 2\nu_1 k_1^2] \bar{\beta}_2^{(1)} = 0 \quad (i)$$

$$\kappa_n^2 \sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)}) + 2[(1 + \nu_2) \kappa_n^2 + \nu_2 k_1^2] \bar{\beta}_2^{(2)}$$

$$+ 2[(1 + \nu_1) \kappa_n^2 + \nu_1 k_1^2] \bar{\beta}_2^{(1)} = -\frac{i \xi_n k_1 [2Q_c k_1^2 + (Q_b + 2Q_c) \kappa_n^2]}{k_1^2 + \kappa_n^2} \quad (j)$$

$$\Delta \sigma_{12}^{A_n V} = \Delta \sigma_{12}^{A_n \infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)}) + 2(\nu_2 \bar{\beta}_2^{(2)} + \nu_1 \bar{\beta}_2^{(1)}) = -\frac{i \xi_n [Q_b k_1^2 + (Q_b - Q_c) \kappa_n^2]}{k_1 (k_1^2 + \kappa_n^2)} \quad (k)$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)}) + (1 + 2\nu_2) \bar{\beta}_2^{(2)} - (1 + 2\nu_1) \bar{\beta}_2^{(1)} = 0 \quad (l)$$

$$\Delta \sigma_{13}^{A_n V} = \Delta \sigma_{13}^{A_n \infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)}) + \bar{\beta}_2^{(2)} - \bar{\beta}_2^{(1)} = 0 \quad (m)$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)}) + 2(\bar{\beta}_2^{(2)} + \bar{\beta}_2^{(1)}) = \frac{i \xi_n [(Q_c - 2Q_b) k_1^2 + (Q_c - Q_b) \kappa_n^2]}{k_1 (k_1^2 + \kappa_n^2)} \quad (n)$$

$$\Delta \sigma_{23}^{A_n V} = \Delta \sigma_{23}^{A_n \infty} \Rightarrow$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} - \bar{\alpha}_2^{(1)}) + 2(\nu_2 \bar{\beta}_2^{(2)} + \nu_1 \bar{\beta}_2^{(1)}) = -\frac{iQ_c \xi_n k_1}{k_1^2 + \kappa_n^2} \quad (o)$$

$$\sqrt{k_1^2 + \kappa_n^2} (\bar{\alpha}_2^{(2)} + \bar{\alpha}_2^{(1)}) + (1 + 2\nu_2) \bar{\beta}_2^{(2)} - (1 + 2\nu_1) \bar{\beta}_2^{(1)} = 0 \quad (p) \quad (14)$$

Next, we are concerned with satisfying boundary conditions restricting ourselves to conditions (13).

This leads to the displacement and stress fields due to an interface edge dislocation ( $\vec{b} = (0, b, 0)$ ) parallel to the  $x_3$  – direction at the origin; the interface is the  $ox_1x_3$  – plane. The elastic fields (to first order in  $A_n(x_3)$ ) for the sinusoidal edge dislocation require additional works using (14). This will be the subject of a separate paper.

### III - CALCULATION RESULTS

#### III-1. Partial elastic fields associated with interface boundary conditions

Three distinct couples of values for  $(\bar{\alpha}_2^{(m)}, \bar{\beta}_2^{(m)})$  are extracted from (13); these are:

$$\begin{aligned}
 \text{(a)} \quad \bar{\alpha}_2^{(m)} &= \frac{\nu_m C_m Q_a}{k_1^3} \equiv \bar{\alpha}_{2a}^{(m)}; \quad \bar{\beta}_2^{(m)} = (-1)^{m-1} \frac{C_m Q_a \operatorname{sgn}(k_1)}{2 k_1^2} \equiv \bar{\beta}_{2a}^{(m)} \\
 \text{(b)} \quad \bar{\alpha}_2^{(m)} &= \frac{2\nu_m Q_b}{k_1^3} \equiv \bar{\alpha}_{2b}^{(m)}; \quad \bar{\beta}_2^{(m)} = (-1)^{m-1} Q_b \frac{\operatorname{sgn}(k_1)}{k_1^2} \equiv \bar{\beta}_{2b}^{(m)} \\
 \text{(c)} \quad \bar{\alpha}_2^{(m)} &= \frac{\bar{V}_c}{k_1^3} \equiv \bar{\alpha}_{2c}^{(m)}, \text{ independent of } m; \quad \bar{\beta}_2^{(m)} = (-1)^{m-1} \frac{Q_c \operatorname{sgn}(k_1)}{\nu_m k_1^2} \equiv \bar{\beta}_{2c}^{(m)} \quad (15)
 \end{aligned}$$

where

$$Q_a = i(\nu_1 - \nu_2) / [(1 - 2\nu_2)(1 - \nu_1) + (1 - 2\nu_1)(1 - \nu_2)],$$

$$\bar{V}_c = Q_b + 2Q_c [1 - (\nu_1 + \nu_2) / 4\nu_1\nu_2].$$

None of these couples satisfies the entire conditions (13). For each couple, we display below the associated elastic fields  $\vec{u}^{(0)(m)V}$  and  $(\sigma)^{(0)(m)V}$  defined in Section 2.2. A superposition of these partial fields will provide the complete form of solution. The couple  $(\bar{\alpha}_{2a}^{(m)}, \bar{\beta}_{2a}^{(m)})$  is obtained from (13 a to d) associated with the displacement. We have at position  $\vec{x} = (x_1, x_2, x_3)$  ( $\vec{u}^{(0)(m)V} \equiv \vec{u}_a^{(0)(m)V}, (\sigma)^{(0)(m)V} \equiv (\sigma)_a^{(0)(m)V}$ )

$$\begin{aligned}
 u_{1a}^{(0)(m)V} &= \frac{iC_m Q_a}{2\mu_m} \left( (-1)^{m-1} (1 - 2\nu_m) \ln |x_1| \delta_A(x_2) + \frac{x_2^2}{r^2} \right), \\
 u_{2a}^{(0)(m)V} &= \frac{ibQ_a}{2\pi} \left( -\tan^{-1} \left( \frac{x_1}{|x_2|} \right) + (-1)^m \frac{1}{2(1 - \nu_m)} \frac{x_1 x_2}{r^2} \right),
 \end{aligned}$$

$$\begin{aligned}
\sigma_{11a}^{(0)(m)V} &= iC_m Q_a \left( (-1)^{m-1} \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right] - 2 \operatorname{sgn}(x_2) \frac{x_1 x_2^2}{r^4} \right), \\
\sigma_{22a}^{(0)(m)V} &= iC_m Q_a \left( (-1)^{m-1} \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right] + 2 \operatorname{sgn}(x_2) \frac{x_1 x_2^2}{r^4} \right), \\
\sigma_{33a}^{(0)(m)V} &= (-1)^{m-1} 2i\nu_m C_m Q_a \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right], \\
\sigma_{12a}^{(0)(m)V} &= (-1)^m iC_m Q_a \frac{x_2(x_2^2 - x_1^2)}{r^4}; \\
\bar{J}_1 &= \int_0^\infty \sin k_1 x_1 dk_1.
\end{aligned} \tag{16}$$

Here,  $\delta_A$  has the following definition:  $\delta_A(x_2) = 0$  when  $x_2 \neq 0$  and  $\delta_A(x_2) = 1$  when  $x_2 = 0$ ;  $r^2 = x_1^2 + x_2^2$ ; Constant terms are omitted in the displacement. It can easily be verified that  $u_{1a}^{(0)(m)} \equiv u_{1a}^{(0)(m)\infty} - u_{1a}^{(0)(m)V}$  have the equal expression with  $m$  on the interface  $x_2 = 0$ ; but the stresses  $\sigma_{12a}^{(0)(m)} \equiv \sigma_{12a}^{(0)(m)\infty} - \sigma_{12a}^{(0)(m)V}$  and  $\sigma_{22a}^{(0)(m)}$  defined in a similar manner exhibit different factors (with  $m$ ) that multiply the equal spatial functions  $x_2(x_1^2 - x_2^2)/r^4$  and  $x_1(x_1^2 + 3x_2^2)/r^4$  respectively. The pair  $(\bar{\alpha}_{2b}^{(m)}, \bar{\beta}_{2b}^{(m)})$  is obtained using (13 e to h) associated with the stresses. This gives (using the similar notations)

$$\begin{aligned}
u_{1b}^{(0)(m)V} &= \frac{iQ_b}{\mu_m} \left( (-1)^{m-1} (1 - 2\nu_m) \ln |x_1| \delta_A(x_2) + \frac{x_2^2}{r^2} \right), \\
u_{2b}^{(0)(m)V} &= \frac{iQ_b}{\mu_m} \left( -2(1 - \nu_m) \tan^{-1} \left( \frac{x_1}{|x_2|} \right) + (-1)^m \frac{x_1 x_2}{r^2} \right), \\
\sigma_{11b}^{(0)(m)V} &= 2iQ_b \left( (-1)^{m-1} \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right] - 2 \operatorname{sgn}(x_2) \frac{x_1 x_2^2}{r^4} \right), \\
\sigma_{22b}^{(0)(m)V} &= 2iQ_b \left( (-1)^{m-1} \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right] + 2 \operatorname{sgn}(x_2) \frac{x_1 x_2^2}{r^4} \right), \\
\sigma_{33b}^{(0)(m)V} &= (-1)^{m-1} 4i\nu_m Q_b \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right], \\
\sigma_{12b}^{(0)(m)V} &= (-1)^m 2iQ_b \frac{x_2(x_2^2 - x_1^2)}{r^4}.
\end{aligned} \tag{17}$$

It can be verified that  $u_{1b}^{(0)(m)} \equiv u_{1b}^{(0)(m)\infty} - u_{1b}^{(0)(m)V}$  are not identical with  $m$  on the interface; but the stresses  $\sigma_{12b}^{(0)(m)} \equiv \sigma_{12}^{(0)(m)\infty} - \sigma_{12b}^{(0)(m)V}$  and  $\sigma_{22b}^{(0)(m)}$  (similar notations) exhibit the identical expressions with  $m$  ( $= 1$  and  $2$ ) with the equal factor  $(C_1 + C_2) / 2$  multiplying the spatial functions  $x_2(x_1^2 - x_2^2) / r^4$  and  $x_1(x_1^2 + 3x_2^2) / r^4$  respectively. The couple  $(\bar{\alpha}_{2c}, \bar{\beta}_{2c}^{(m)})$  is calculated using (13  $g$  to  $j$ ) associated with the stresses. This leads (similar notations apply)

$$\begin{aligned}
 u_{1c}^{(0)(m)V} &= \frac{i}{\nu_m \mu_m} \left( (-1)^m (\nu_m \bar{V}_c - Q_c) \ln |x_1| \delta_A(x_2) + Q_c \frac{x_2^2}{r^2} \right), \\
 u_{2c}^{(0)(m)V} &= -\frac{i}{\nu_m \mu_m} \left( [2(1 - 2\nu_m) Q_c + \nu_m \bar{V}_c] \tan^{-1} \left( \frac{x_1}{|x_2|} \right) + (-1)^{m-1} Q_c \frac{x_1 x_2}{r^2} \right), \\
 \sigma_{11c}^{(0)(m)V} &= \frac{2i}{\nu_m} \left( (-1)^{m-1} [(1 + 2\nu_m) Q_c - \nu_m \bar{V}_c] \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right] \right. \\
 &\quad \left. - 2Q_c \operatorname{sgn}(x_2) \frac{x_1 x_2^2}{r^4} \right), \\
 \sigma_{22c}^{(0)(m)V} &= \frac{2i}{\nu_m} \left( (-1)^{m-1} [(1 - 2\nu_m) Q_c + \nu_m \bar{V}_c] \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right] \right. \\
 &\quad \left. + 2Q_c \operatorname{sgn}(x_2) \frac{x_1 x_2^2}{r^4} \right), \\
 \sigma_{33c}^{(0)(m)V} &= (-1)^{m-1} 4i Q_c \left[ \frac{x_1}{r^2} + \left( \bar{J}_1 - \frac{x_1}{r^2} \right) \delta_A(x_2) \right], \\
 \sigma_{12c}^{(0)(m)V} &= \frac{2ix_2}{r^2} \left( (2Q_c - \bar{V}_c) \operatorname{sgn}(x_2) + (-1)^m \frac{Q_c}{\nu_m} \frac{(x_2^2 - x_1^2)}{r^2} \right) \\
 &\quad + 2\pi i (2Q_c - \bar{V}_c) \delta(x_1) \delta_A(x_2)
 \end{aligned} \tag{18}$$

where  $\delta(x_1)$  is the well-known Dirac delta function in  $\sigma_{12c}^{(0)(m)V}$  when  $x_2 = 0$ .

### III-2. Displacement and stress fields due to an interface straight edge dislocation

#### III-2-1. Boundary conditions

We define the elastic fields  $\bar{u}^{(0)(m)}(\bar{x})$  and  $(\sigma)^{(0)(m)}(\bar{x})$  as

$$\begin{aligned}\bar{u}^{(0)(m)} &= \bar{u}^{(0)(m)\infty} - \bar{u}^{(0)(m)W} \\ (\sigma)^{(0)(m)} &= (\sigma)^{(0)(m)\infty} - (\sigma)^{(0)(m)W}\end{aligned}\quad (19)$$

with

$$\begin{aligned}\bar{u}^{(0)(m)W} &= \eta_a^{(m)} \bar{u}_a^{(0)(m)V} + \eta_b^{(m)} \bar{u}_b^{(0)(m)V} + \eta_c^{(m)} \bar{u}_c^{(0)(m)V} . \\ (\sigma)^{(0)(m)W} &= \eta_a^{(m)} (\sigma)_a^{(0)(m)V} + \eta_b^{(m)} (\sigma)_b^{(0)(m)V} + \eta_c^{(m)} (\sigma)_c^{(0)(m)V}\end{aligned}\quad (20)$$

Again  $\bar{u}^{(0)(m)\infty}$  and  $(\sigma)^{(0)(m)\infty}$  are due to a straight edge dislocation ( $\bar{b} = (0, b, 0)$ ) parallel to the  $x_3$  - direction at the origin in an infinitely extended homogeneous medium ( $m$ ) (see Anongba[1, 4, 5]);  $\bar{u}_{a \text{ to } c}^{(0)(m)V}$  and  $(\sigma)_{a \text{ to } c}^{(0)(m)V}$  are given in (16) to (18);  $\eta_{a \text{ to } c}^{(m)}$  are real to be determined by the requirement that the elastic fields satisfy the following conditions :

- $\bar{u}^{(0)(m)}(\bar{x})$  is continuous across the interface (actually we shall write this condition for the  $x_1$  - component).
- $\oint_{\Gamma} du_2^{(0)(m)} = b$  for a closed contour in  $x_1 x_2$  encircling the dislocation. We may take for  $\Gamma$  a square of side  $a$  centred at the origin and travelled in the direction of the corkscrew advancing in the positive  $x_3$  - direction.
- The stresses  $\sigma_{ij}^{(0)(m)}$  are continuous at the crossing of the interface, i.e.  $\sigma_{ij}^{(0)(1)}(x_1, x_2 = 0, x_3) = \sigma_{ij}^{(0)(2)}(x_1, x_2 = 0, x_3)$ .
- $\bar{u}^{(0)(m)W}(\bar{x})$  vanish far from the interface (i.e. when  $|x_2| \rightarrow \infty$ ).

It can be seen that all the stresses involved in  $(\sigma)^{(0)(m)\infty}$  and  $(\sigma)_{a \text{ to } c}^{(0)(m)V}$  vanish at infinity. Under such conditions above,  $\bar{u}^{(0)(m)}(\bar{x})$  and  $(\sigma)^{(0)(m)}(\bar{x})$  correspond to an interface straight edge dislocation. Next, we express the quantities involved in the conditions above and proceed to satisfy these.

$$\begin{aligned}u_1^{(0)(1)}(x_1, x_2 = 0, x_3) &= u_1^{(0)(2)}(x_1, x_2 = 0, x_3) \Rightarrow \\ \frac{1 - 2\nu_m}{2\mu_m} &\left\{ C_m - \eta_a^{(m)} (-1)^{m-1} C_m i Q_a - \eta_b^{(m)} (-1)^{m-1} 2i Q_b \right. \\ &\left. - \eta_c^{(m)} (-1)^{m-1} \frac{2i(Q_c - \nu_m \bar{V}_c)}{\nu_m (1 - 2\nu_m)} \right\} \equiv e_1 ;\end{aligned}$$



$$\int_{\Gamma} du_2^{(0)(m)} = b \Rightarrow$$

$$\eta_a^{(m)} iQ_a + \eta_b^{(m)} \frac{2iQ_b}{C_m} + \eta_c^{(m)} \frac{2\pi i}{b\mu_m} \left( \frac{2(1 - 2\nu_m)Q_c}{\nu_m} + \bar{V}_c \right) \equiv e_2 ;$$

$$\sigma_{22}^{(0)(1)} = \sigma_{22}^{(0)(2)} \Rightarrow$$

$$C_m - \eta_a^{(m)} (-1)^{m-1} C_m iQ_a - \eta_b^{(m)} (-1)^{m-1} 2iQ_b$$

$$- \eta_c^{(m)} (-1)^{m-1} 2i \left( \frac{(1 - 2\nu_m)Q_c + \nu_m \bar{V}_c}{\nu_m} \right) \equiv e_3 ;$$

$$\sigma_{12}^{(0)(1)} = \sigma_{12}^{(0)(2)} \Rightarrow$$

$$\eta_c^{(m)} = \eta_c \equiv e_4 ;$$

$$\sigma_{33}^{(0)(1)} = \sigma_{33}^{(0)(2)} \Rightarrow$$

$$2\nu_m \left\{ C_m - \eta_a^{(m)} (-1)^{m-1} C_m iQ_a - \eta_b^{(m)} (-1)^{m-1} 2iQ_b - \eta_c^{(m)} (-1)^{m-1} 2i \frac{Q_c}{\nu_m} \right\} \equiv e_5 ;$$

$$\sigma_{11}^{(0)(1)} = \sigma_{11}^{(0)(2)} \Rightarrow$$

$$C_m - \eta_a^{(m)} (-1)^{m-1} C_m iQ_a - \eta_b^{(m)} (-1)^{m-1} 2iQ_b$$

$$- \eta_c^{(m)} (-1)^{m-1} 2i \left( \frac{(1 + 2\nu_m)Q_c - \nu_m \bar{V}_c}{\nu_m} \right) \equiv e_6 ;$$

$\bar{u}^{(0)(m)W}(\bar{x})$  vanishes when  $|x_2| \rightarrow \infty \Rightarrow$

$$\eta_a^{(m)} \nu_m C_m iQ_a - \eta_b^{(m)} \nu_m 2iQ_b + \eta_c^{(m)} 2iQ_c = 0 \equiv e_7 ; \tag{21}$$

where all  $e_i$  are constant with  $m = 1$  and  $2$  ;for the various stress conditions above, we restrict ourselves to terms with the spatial function  $(1 / x_1)$  only.

**III-2-2. Satisfying boundary conditions**

We are concerned with finding the appropriate expressions for  $\eta_{a \text{ to } c}^{(m)}$  that satisfy the boundary conditions (21). We first recognized that  $\sigma_{12}^{(0)(m)}$  (19) takes on the interface the very simple form involving only  $\eta_c^{(m)}$

$$\sigma_{12}^{(0)(m)}(x_1, x_2 = 0, x_3) = -\eta_c^{(m)} 2\pi i (2Q_c - \bar{V}_c) \delta(x_1). \quad (22)$$

$\sigma_{12}^{(0)(1)} = \sigma_{12}^{(0)(2)}$  on the interface leads to  $\eta_c^{(m)} = \eta_c \equiv e_4$  constant with  $m$  (21). A number of expressions for  $\eta_c^{(m)}$  can be extracted from (21), but only one value leaves  $\sigma_{12}^{(0)(m)}$  (22) unchanged on inverting the elastic constants. It is obtained as

$$\eta_c = \frac{2b\mu_1\mu_2[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)]}{[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)]2\pi i(2Q_c - \bar{V}_c)}. \quad (23)$$

This can be arrived at from different routes involving different equations in (21). Only one route is presented as follows.  $\eta_c^{(2)}$  can be isolated using (1)  $e_3$  and  $e_6$  in (21), (2)  $e_1$  and  $e_3$  and, (3)  $e_1$  and  $e_2$ ; (1)=(2) and (1)=(3) provide, respectively, the following **Equations** :

$$C_1 - \eta_a^{(1)} C_1 iQ_a - \eta_b^{(1)} 2iQ_b + \eta_c \frac{2i\{v_1(\mu_2 - \mu_1)\bar{V}_c + [-\mu_2 + \mu_1(1 + 2v_1 - 2v_2)]Q_c\}}{v_1[\mu_2(1 - 2v_1) - \mu_1(1 - 2v_2)]} = 0 \quad (24)$$

$$2(1 - v_1)(1 - 2v_2)C_1 - b_1^*(C_1 - \eta_a^{(1)} C_1 iQ_a - \eta_b^{(1)} 2iQ_b) + \eta_c \frac{i}{v_1\mu_2} (2Q_c b_2^* - v_1 \bar{V}_c [\mu_2 - \mu_1(3 - 4v_2)]) = 0 \quad (25)$$

where

$$b_1^* = (1 - v_1)(1 - 2v_2) + (1 - v_2)(1 - 2v_1),$$

$$b_2^* = \mu_2(2 - 2v_1 - 3v_2 + 4v_1v_2) - \mu_1v_1(3 - 4v_2).$$

Both **Equations** (24) and (25) yield  $\eta_c$  (23). Using (23), **Equations** (24) and  $e_7$  (21) allow  $\eta_a^{(1)}$  and  $\eta_b^{(1)}$  to be calculated; then to reach  $\eta_a^{(2)}$  and  $\eta_b^{(2)}$ , we may associate to  $e_7$  (21), the following equation (26) obtained in the same way as in (24) :

$$C_2 + \eta_a^{(2)} C_2 iQ_a + \eta_b^{(2)} 2iQ_b + \eta_c \frac{2i\{v_2(\mu_2 - \mu_1)\bar{V}_c + [\mu_1 - \mu_2(1 - 2v_1 + 2v_2)]Q_c\}}{v_2[\mu_1(1 - 2v_2) - \mu_2(1 - 2v_1)]} = 0. \quad (26)$$

We have

$$\eta_b^{(1)} - \eta_b^{(2)} = \frac{C_1 + C_2}{C_1 - C_2} = \frac{\mu_1(1 - \nu_2) + \mu_2(1 - \nu_1)}{\mu_1(1 - \nu_2) - \mu_2(1 - \nu_1)}$$

$$\eta_b^{(2)} = \frac{1}{C_2 - C_1} \left( C_2 + \frac{2b\mu_1\mu_2(\mu_1 - \mu_2)}{\pi[\mu_1^2(3 - 4\nu_2) - \mu_2^2(3 - 4\nu_1)]} \right). \quad (27)$$

$\eta_a^{(m)}$  depends on elastic constants, this is denoted as  $\eta_a^{(m)} = \eta_a^{(m)}(\nu_1, \nu_2; \mu_1, \mu_2)$ ; by inverting the elastic constants, this becomes  $\eta_a^{(m)}(\nu_2, \nu_1; \mu_2, \mu_1)$ . We have the following results:

$$\eta_a^{(1)}(\nu_1, \nu_2; \mu_1, \mu_2) + \eta_a^{(2)}(\nu_2, \nu_1; \mu_2, \mu_1) = -\frac{i}{Q_a};$$

$$\eta_a^{(2)}(\nu_1, \nu_2; \mu_1, \mu_2) = \frac{1}{2iQ_a} \left( -1 + \frac{Nu}{De} \right) \quad (28)$$

where

$$Nu = 2b\mu_1(\nu_1(1 - \nu_2)d_1^*\mu_1^2 + \nu_2(1 - \nu_1)e_2^*\mu_2^2 + \mu_1\mu_2[\nu_1(1 - \nu_2)e_1^* + \nu_2(1 - \nu_1)d_2^*])$$

$$De = \pi(1 - \nu_1)(\nu_1 - \nu_2)(\nu_1C_1 + \nu_2C_2)[\mu_1^2(3 - 4\nu_2) - \mu_2^2(3 - 4\nu_1)]$$

in which

$$d_1^* = \nu_2 - 5\nu_1 + 8\nu_1\nu_2; \quad d_2^* = \nu_2 + 3\nu_1 - 8\nu_1\nu_2,$$

$$e_1^* = 5\nu_1 - \nu_2 - 8\nu_1^2; \quad e_2^* = -3\nu_1 - \nu_2 + 8\nu_1^2.$$

In summary,  $\eta_{a \text{ to } c}^{(m)}$  are determined by using (23), (27 and 28).

### III-2-3. Perfect elastic fields

The elastic fields due to an interface straight edge dislocation ( $\vec{b} = (0, b, 0)$ ) parallel to the  $x_3$  - direction at the origin, are given in (19 and 20) with values of  $\eta_{a \text{ to } c}^{(m)}$  calculated from (23), (27 and 28), respectively. We display only the special values taken on the interface by quantities  $\sigma_{12}^{(0)(m)}$  and  $\sigma_{22}^{(0)(m)}$  that are frequently involved in the analyses of the propagation of the interface crack loaded in tension.

$\sigma_{12}^{(0)(m)}$  is obtained from (22 and 23) as

$$\sigma_{12}^{(0)(m)}(x_1, x_2 = 0, x_3) = - \frac{2b\mu_1\mu_2[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)]}{[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)]} \delta(x_1), \quad (29)$$

$m = 1$  and  $2$ . This quantity is unchanged by inverting the elastic constants. From (21),  $\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3)$  is first written as

$$\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3) = \frac{e_3}{x_1}. \quad (30)$$

The calculation can be performed with  $m=1$ . Introducing in (30) the value of  $(C_1 - \eta_a^{(1)} C_1 iQ_a - \eta_b^{(1)} 2iQ_b)$  taken from (24) and making use of the value of  $\eta_c$  (23), we obtain

$$e_3 = \frac{\bar{N}u}{\bar{D}e} \quad (31)$$

$$\bar{N}u = 4b\mu_1\mu_2(\nu_1(1-\nu_2)a_1^*\mu_1^2 + \nu_2(1-\nu_1)a_2^*\mu_2^2 + \mu_1\mu_2[-(\nu_1-\nu_2)^2 + 4\nu_1\nu_2a_3^*])$$

$$\bar{D}e = \pi(\nu_1-\nu_2)[\mu_1^2(3-4\nu_2) - \mu_2^2(3-4\nu_1)][\nu_1(1-\nu_2)\mu_1 + \nu_2(1-\nu_1)\mu_2]$$

where

$$a_1^* = \nu_1 - \nu_2 - 4\nu_1\nu_2^2; \quad a_2^* = \nu_2 - \nu_1 - 4\nu_2\nu_1^2, \quad a_3^* = \nu_1^2 + \nu_2^2 - \nu_1\nu_2^2 - \nu_1^2\nu_2.$$

$e_3$  (31) is unchanged by inverting the elastic constants.

#### IV - DISCUSSION

Expressions for the displacement and stress fields of interface straight edge dislocations with their associated Airy stress functions have been given [12,13]. In the geometry of the **Figure 1**, the calculated shear stress  $\sigma_{12}(x_1, x_2 = 0, x_3)$  is zero on the interface for an edge dislocation with Burgers vector  $\bar{b} = (0, b, 0)$  perpendicular to the interface. Later on, [14] has stressed that this shear stress contains a term proportional to the Dirac delta function; this result has been incorporated in a number of analyses of the propagation of the interface crack under load [15-18].

Incorporating a Dirac delta function in the value of the shear stress on the interface is a clear indication that the elasticity solutions given by [12] are

partial. In the present study, a Dirac delta function is present in the complete elastic fields (Section 3.2.3) and pertains to the partial elastic fields  $\vec{u}_c^{(0)(m)V}$   $(\sigma)_c^{(0)(m)V}$  only (see (18), Section 3.1). This suggests that the results of the present study are potentially more general; however, discrepancies between our expressions for  $\sigma_{12}^{(0)(m)}(x_1, x_2 = 0, x_3)$  (29) and  $\sigma_{22}^{(0)(m)}(x_1, x_2 = 0, x_3)$  (30 and 31) and those given below by [16] are observed. In the geometry of the **Figure 1**, their results are (solid S1):

$$\sigma_{12}^{(1)}(x_1, x_2 = 0, x_3) = - \frac{2b\mu_1\mu_2[\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)]}{[\mu_1 + \mu_2(3-4\nu_1)][\mu_2 + \mu_1(3-4\nu_2)]} \delta(x_1);$$

$$\sigma_{22}^{(1)}(x_1, x_2 = 0, x_3) = \frac{4b\mu_1\mu_2[\mu_1(1-2\nu_2) + \mu_2(1-2\nu_1)]}{\pi[\mu_1 + \mu_2(3-4\nu_1)][\mu_2 + \mu_1(3-4\nu_2)]} \frac{1}{x_1}.$$

Their shear stress  $\sigma_{12}^{(1)}$  changes sign on inverting the elastic constants indicating that it is discontinuous on crossing the interface from solid S1 ( $\sigma_{12}^{(1)}$ ) to solid S2 ( $\sigma_{12}^{(2)}$ ). In contrast our result for  $\sigma_{12}^{(0)(m)}$  (29) is continuous at the crossing of the interface. This indicates an essential difference between the interface boundary conditions in both studies. Their  $\sigma_{22}^{(1)}$  above is continuous across the interface like our value (30 and 31). A step in the road toward the elastic fields due to interface sinusoidal edge dislocations (**Figure 1**) requires satisfying boundary conditions involving the interface boundary conditions (14) corresponding to linear expressions with respect to the perturbation  $A_n(x_3)$ . This will be the subject of a separate paper.

## V - CONCLUSION

In the present study, Galerkin vectors have been used (Section 2.2) and associated interface boundary conditions have been displayed. These conditions can be decomposed into two groups: (1) the first group corresponds to a planar interface with a straight edge dislocation at the origin (13); (2) the second group corresponds to terms proportional to the sinusoid or its spatial derivative with respect to  $x_3$  in the elastic fields expressed to first order with respect to the perturbation  $A_n(x_3) = \xi_n \sin \kappa_n x_3$  (14). Satisfying both boundary conditions leads to terms of first order with respect to  $\xi_n$  in the elastic fields of a sinusoidal edge dislocation (see **Figure 1**).

In the present paper, we have restricted ourselves to satisfying (13) only. Expressions of the displacement and stress fields of interface straight edge

dislocation have been thus obtained. Our results contain the Dirac delta function in the shear stresses on the interface. We have also compared our findings with those previously published on similar problems.

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