

## PROPAGATION OF TIDAL WAVES IN THE SAN PEDRO HARBOR- WEST COAST OF COTE D'IVOIRE

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### ABSTRACT

San Pedro harbor is a semi-enclosed port with an average depth of 6 m, opened only to the Atlantic Ocean. The tidal wave runs from the dock of Servitude to the fishing port with decreasing amplitude and a speed of 0.9 m/s approximately. Between the two stations, a time shift difference of 19 minutes is observed between two tides heights.

**Keywords :** *Tidal wave, height, speed, port, San Pedro*

### RÉSUMÉ

**Propagation de l'onde de marée dans le port de San-Pedro, côte ouest de la Côte d'Ivoire.**

Le port de San Pedro est un port semi-fermé ayant une profondeur moyenne de 6 m ouvert uniquement à l'océan Atlantique. L'onde de marée se propage du quai de servitude au port de pêche avec une amplitude qui décroisse et une vitesse d'environ 0,9 m/s. Entre les deux stations, on observe un décalage horaire de 19 minutes entre deux hauteurs de marées.

**Mots-clés :** *Onde de marée, hauteur, vitesse, port, San Pedro*

## I - INTRODUCTION

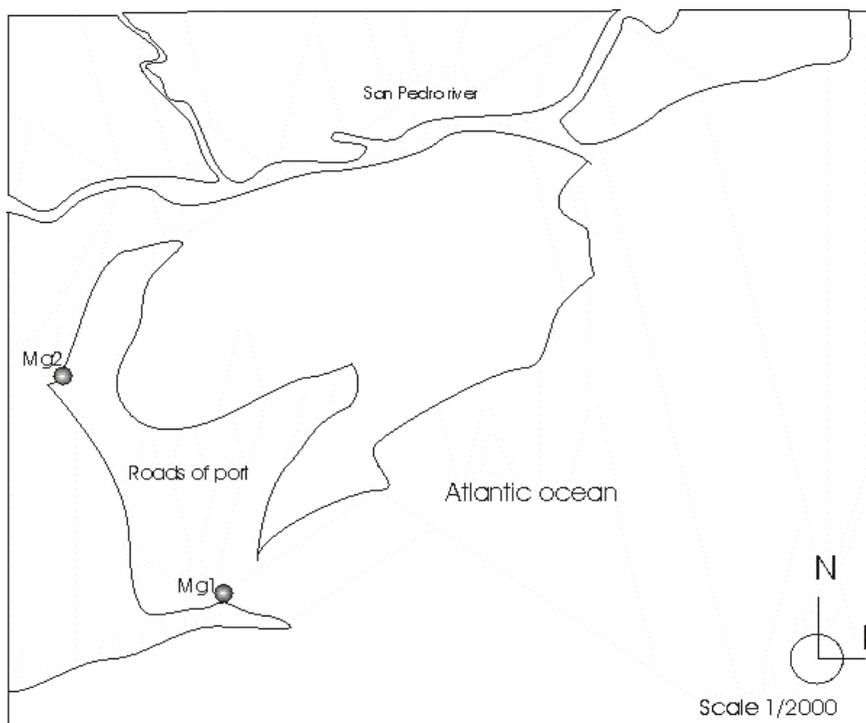
The tide is defined like the upswing then going down from water of the oceans caused by the combined effect of the forces of gravitation of the Moon and the Sun. The tidal wave which is followed from there propagates with a speed and an amplitude which decrease along the rivers, rivers, estuaries where ports.

The object of this study is to determine the characteristics of the tidal wave and its influence on the structures of the harbor roads of San Pedro.

## II - MATERIEL AND METHODS

### II-1. Presentation of the study area

San Pedro harbor is located in the South-west part of Côte d'Ivoire near the Atlantic Ocean, where was the previous inlet of the river San Pedro (*Figure 1*). This port was created in 1978, to diminish the congestion of the Abidjan port due to agricultural products such as cocoa, coffee and wood from the western and South-western area of the country [1]. This port seems to play its role in providing goods to the South-western, Western and Northern part of the country.



**Figure 1** : Locations of tide-gauges in San Pedro harbor [2].

Studies showed that the tidal waves are propagated along the estuaries and go up the course of rivers when their topography is sufficiently low [3-5]. Under these conditions, most of coastal harbors are influenced by tide, which is known as vertical oscillatory movements of the sea surface caused by tidal waves passing through.

## II-2. Force generating tide

According to Newton, in oceans, tide generating forces obey the laws of gravitational attraction in the Sun-Earth-Moon system.

When considering the system Earth-Moon, with the moon at the equator, any water body in the ocean will undergo the gravity force and two other forces: a centrifugal force and an attraction force to the moon. The centrifugal force is the same all over the earth, but the attraction force to the moon is stronger in the closest side of the moon than on the opposite side. If the world were not covered with continent, the resulting force would produce a high tide on the moon side, a high tide on the opposite side and low tides at the poles [6]. These movements will follow the moon in its rotation around the Earth. These “theoretical” tides are called the tides of balance [7].

The generating force of the lunar tide of balance is given by:

$$FGM = 3/2 * M/E * (a/d)^3 * g * 8 * \sin(2\theta)$$

Where:

M = mass of the Moon

E = mass of the Earth

a = ray of the Earth = 6.37\*10<sup>3</sup> km

d = distance between the centers of the Earth and the Moon = 384\*10<sup>3</sup> km

$\theta$  = angle between the line joining the centers of the Earth and the moon and the line joining the center of the Earth to the water piece to the position P on the surface of the ocean.

This reveals that:

- the distribution of the generating force of the tide varies in space on the Earth;
- this distribution varies in time following the movement of the moon compared to the Earth.

## II-3. Data origin and processing

### II-3-1. Origin of the data

The actual study on tide propagation is based on the data collected from August 28th to September 26th, 2002 in the port of San Pedro. Measurements

of water level resulting from two tide gauges located in the port, separated approximately of 975 m; the first one settled at the quay of constraint (harbor office) and the second one within the fishing harbor (**Figure 2**). Their positions in longitude and latitude are presented in **Table 1**.

**Table 1:** Positions of the tide gauges

Positions	Longitude	latitude
Tide gauge 1	6°36'37' 'W	4°44'00' 'NR
Tide gauge 2	6°63'90' 'W	4°45'22' 'NR

### **II-3-2. Data processing**

#### *II-3-2-1. Filtering water level series*

The time series in general are a superposition of the fluctuations of high frequencies (HF) and low frequencies (BF) (oscillations of long periods). The time series of levels were thus filtered in order to isolate low frequency oscillations. By subtraction of the original series, the high frequency oscillations are then obtained. The operator “filters low-pass “is used to separate the oscillations HF from the BF of series of water levels (**Figure 2 and 3**). The principle of this filter is described by *Walters and Heston* [8].

#### *II-3-2-2. Harmonic Analyze of tide*

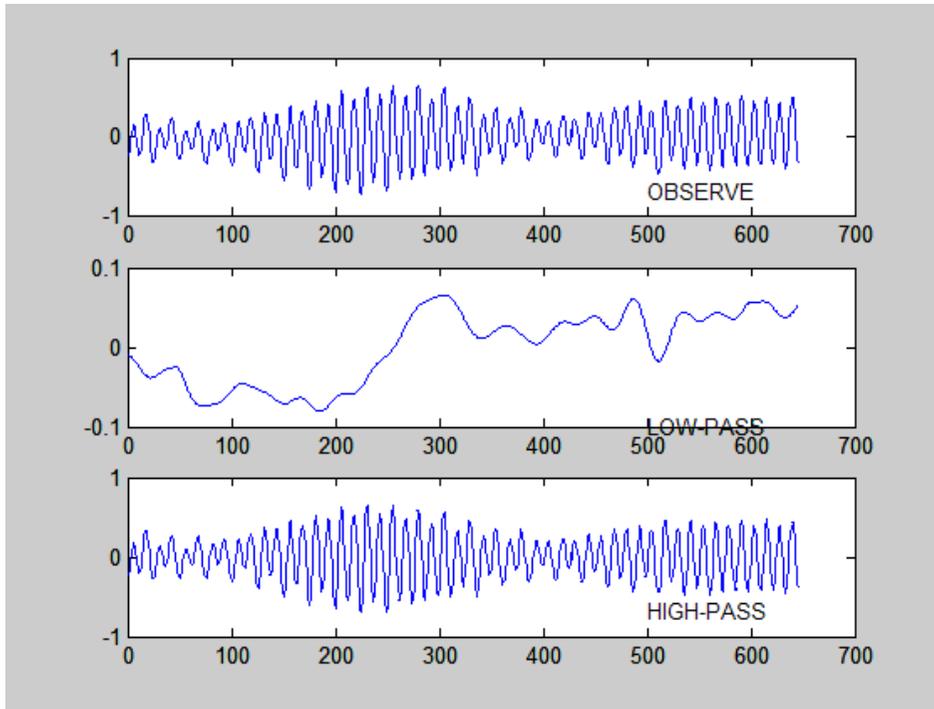
The series of water level were treated by harmonic analysis [9]. This analysis makes it possible to extract from the information of level the deterministic oscillations of astronomical origin whose sum constitutes the signal of tide. The Fortran program Matlab (t\_tide) of Pawlowicz [10; 11], by Foreman [9], was used to carry out the harmonic analysis. This program allows obtaining a prediction of the tide with estimation of errors.

The predicted tide is presented at figure 4 and the results of the harmonic analysis are given by **Table 1**.

## **III - RESULTS AND DISCUSSION**

### **III-1. Low and high frequencies Oscillations**

**Figure 2** shows a juxtaposition of time oscillations observed, low frequencies and high frequencies of the tide recorded with tide gauge 1.



**Figure 2** : Time oscillations (*OBSERVES*), low frequency (*LOW-PASS*) and high frequency (*HIGH-PASS*) of the water level recorded by tide gauge 1. (X-coordinate: frequency – hour; Y-coordinate: level- m)

### ***III-1-1. Low frequency oscillations (BF)***

There is a fall of the water level between August 28 and September 06 (between 80 and 200 hours *LOW-PASS*) at station 1 (**Figure 2**). The low frequency oscillations of the sea level are also the vertical demonstration of hydrodynamic processes acting in the horizontal level. It is the principle of the continuity of the movement. Analysis of these oscillations makes it possible to include/understand, in a certain case, some aspects of the horizontal water circulation. For example, a rise of the atmospheric pressure causes a fall of the level of the effect (barometer reverses effect), drop which induces a volume reduction and thus an horizontal exit of water by the port inlet.

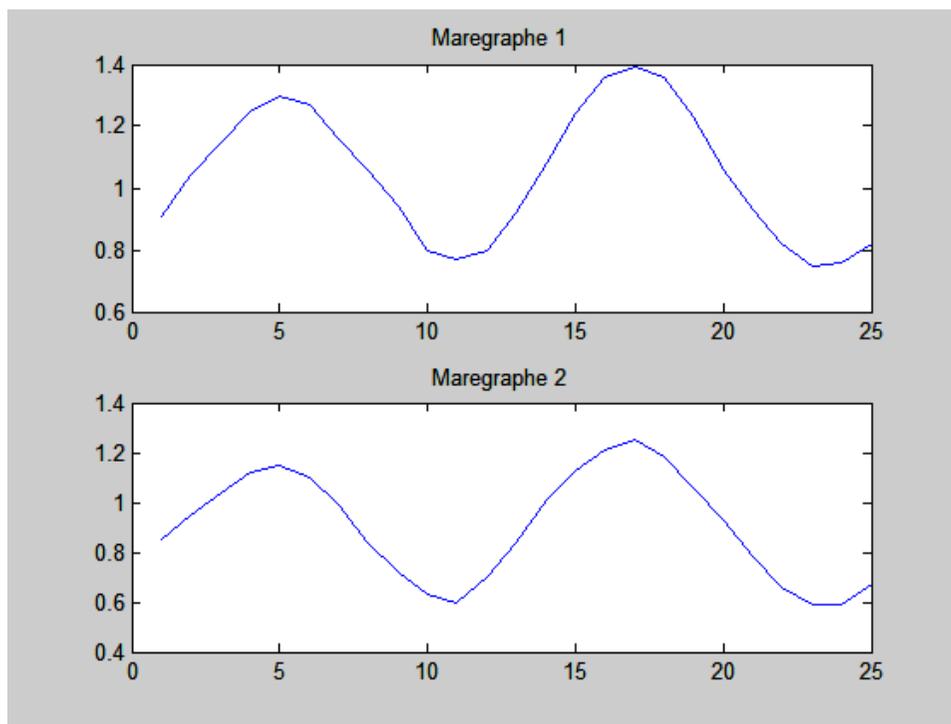
### ***III-1-2. Oscillations of high frequencies (HF)***

It is noted that there is no notable difference between the high frequencies oscillations and the tide observed (**Figure 2**). Therefore, influence of tide is dominant in the port but the presence of other phenomena forcing in the high

frequencies is to be taken into account, because the whole energy is not produced by tide.

### III-2. Water level in the port

Water levels recorded by the two tide gauges of the port are dephasing (**Figure 3**). But the water height is higher on the level of the quay of constraint (tide gauge 1) where the levels are higher than 1.2 m. Within the fishing port (tide gauge 2), levels remain lower or equal to 1.2 m. During this cycle of tide (period of 12 midnight), the second high tide of this period is most important, with a 1.4 m height for tide gauge 1 and 1.2 m for tide gauge 2, are a difference of the water level of 0.2 m between the two tide gauges. Difference in level due to the deceleration of the tidal wave and the reduction of the water depth .



**Figure 3** : water level line of the 2 tide gauges from August 28th to 29th at 4pm. (X-coordinate: length - hour; Y-coordinate: level- m)

A strong increase in the water level inside the port from September 6 at 3 pm, could be due to the pressure decrease in the harbor zone. The level starts

falling at the end of September 12 until September 18, to carry out a light increase from 19 to 26 September, 2002.

The variations of extreme levels of water in the port (**Figure 2**) coincide with the first week of September (where the height reaches 1.7 m) which is the beginning of the small rainy season in the south western part of the country [12]. Indeed, taking into account the proximity of the San Pedro river mouth, the water level in the port increases by the contributions of the river San Pedro during this beginning of rain and especially in the flood period. Moreover this period (September-October) corresponds to the autumnal equinox where tides present a strong marling [13]. The average unevenness at spring tides is approximately 1.5 m, while the unevenness at the time of the average tides is approximately 0.8 m.

Also, inside the San Pedro harbor, and at the level of the channel of approach, ships moored within the port or anchored offshore, will undergo a unevenness varying between 0.8 and 1.5 m, the average height is closed to 1.2 m.

Local fluctuations of the water level, related to the propagation of tidal wave, are accompanied by very strong variations of the flow. Indeed, the rising tide causes upstream temporary storage of volume [14] of water which is then discharged during the low tide, causing fluctuations of the medium flow: when the level increases with the rising tide, the flow decreases very quickly and reaches its minimum when the level is with its apogee.

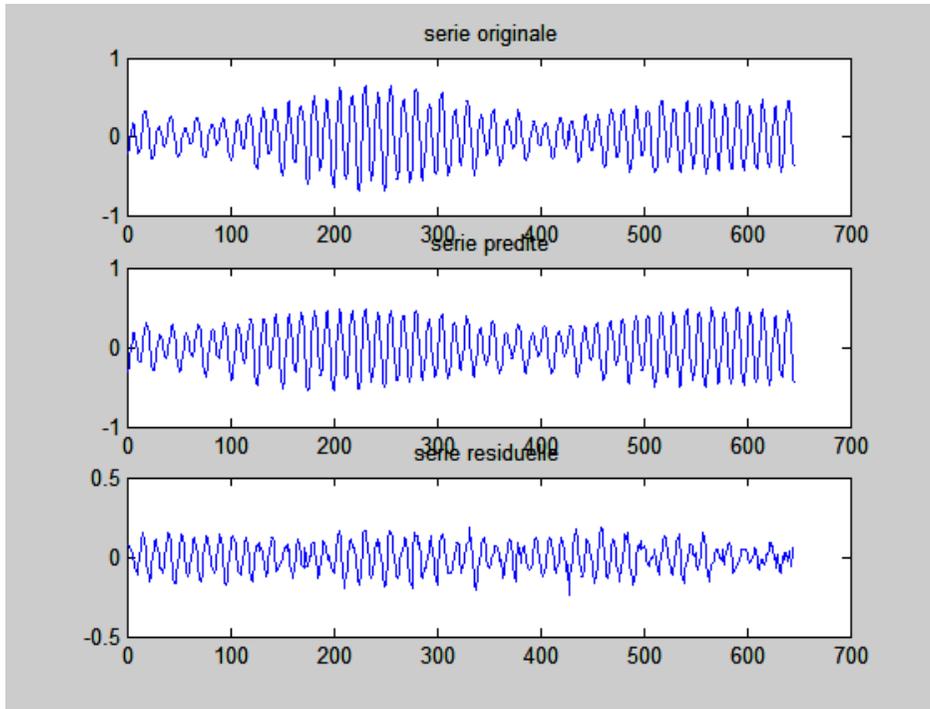
### III-3. Predictions of tide

According to **Figure 4**, tide seems to be the prevalent force on the water level in the port of San Pedro, which confirms the ratio prediction / original data of 88.9% for tide gauge 1 (**Table 2**) and of 90.5% for tide gauge 2 (**Table 3**).

Residual high frequency shows amplitude lower than the one predicted.

The position of the moon compared to the Earth and to the sun produces also variations in the height of the tide. Thus twice per month approximately, the tide will be strong or “spring tide”, when the moon is “full” or “new”, and twice per month the tide will be weak or of “neap tide”, when the moon is with its first or last part.

There is a substantial difference between the successive heights of the tides because of their diurnal harmonic components; the successive heights can also be influenced by weather factors.



**Figure 4** : Time oscillations of the water level (high), prediction of the corresponding tide (center) and residual oscillations (low) of tide gauge 1 (X-coordinate: frequency – hour; Y-coordinate: level- m)

#### III-4. Analyze harmonic tide

**Tables 2** and **3** have the results of the harmonic analysis of the water levels of the two tide gauges. In the port of San Pedro, the tide is mixed [15], e.g. that there are semi-diurnal components of equivalent amplitudes. The harmonic component which shows the greatest amplitude in San Pedro is the semi-diurnal component  $m_2$  of period 12.42 hours or lunar half-day, followed by diurnal components  $S_2$  and  $K_1$  (**Tables 2 and 3**). The relative phase of each harmonic component of the tide compared to spindle GMT is indicated in tables 2 and 3. While comparing the phase of the wave  $m_2$  at the two stations, one realizes that the wave is propagated in the port with an increasing phase (station 1,  $phase_1 = 166.39^\circ$ ; station2,  $phase_2 = 157.76^\circ$ ).

**Table 2 :** *Water level harmonic analyze for the tide gauge 1 with an estimate of 95% ratio of prediction / original data of 88.9%.*

Tide	frequency	amplitude	Amplitude error	phase	Error of phase	Signal-to-noise report/ratio
MSF	0.0028219	0.0005	0	205.48	70.59	1.10
*O1	0.0387307	0.0403	0.016	16.81	26.45	6.40
*P1	0.0415526	0.0344	0.016	7.99	31.20	4,60
*K1	0.0417807	0.1040	0.018	0.92	10.28	34
<b>*M2</b>	<b>0.0805114</b>	<b>0.3488</b>	<b>0.078</b>	<b>166.39</b>	<b>12.92</b>	<b>20</b>
*S2	0.0833333	0.1303	0.074	162.04	35.69	3.10
K2	0.0835615	0.0355	0.049	184.44	105.65	0.53
M3	0.1207671	0.0111	0.012	33.81	63.89	0.92
SK3	0.1251141	0.0100	0.012	103.40	73.85	0.71
M4	0.1610228	0.0038	0.006	173.16	108.53	0.40
MS4	0.1638447	0.0032	0.005	217.06	115.38	0.39
S4	0.1666667	0.0042	0.006	255.12	86.97	0.50
2MK5	0.2028035	0.0026	0.005	244.31	119	0.33
2SK5	0.2084474	0.0030	0.005	212.23	99.15	0.38
M6	0.2415342	0.0022	0.002	271.18	72.01	0.88
2MS6	0.2443561	0.0012	0.002	306.12	116.08	0.29
2SM6	0.2471781	0.0023	0.002	332.17	67.63	0.93
3MK7	0.2833149	0.0023	0.003	235.74	102.69	0.44
M8	0.3220456	0.0019	0.003	340.23	97.52	0.44

**Table 3:** *Harmonic analyze of water level for tide gauge 2 with an estimate of 95% and a ratio prediction /given original of 90.5%.*

Tide	frequency	amplitude	Amplitude error	phase	Error of phase	Signal-to-noise report/ratio
MSF	0.0028219	0.0009	0.001	231.43	107.72	0.48
O1	0.0387307	0.0312	0.033	40.97	70.35	0.92
P1	0.0415526	0.0372	0.036	344.16	52.80	1.10
*K1	0.0417807	0.1125	0.038	337.09	15.85	9
<b>*M2</b>	<b>0.0805114</b>	<b>0.3491</b>	<b>0.084</b>	<b>157.76</b>	<b>13.83</b>	<b>17</b>
*S2	0.0833333	0.1218	0.082	158.74	40.97	2.20
K2	0.0835615	0.0331	0.056	181.14	117.97	0.35
M3	0.1207671	0.0068	0.014	105.97	128.14	0.24
SK3	0.1251141	0.0099	0.014	56.19	101.89	0.52
M4	0.1610228	0.0033	0.006	21.41	132.08	0.27

MS4	0.1638447	0.0018	0.006	16.58	185.50	0.093
S4	0.1666667	0.0034	0.006	264.25	137.13	0.33
2MK5	0.2028035	0.0032	0.006	320.88	116.34	0.27
2SK5	0.2084474	0.0079	0.009	136.60	61.22	0.82
M6	0.2415342	0.0033	0.005	172.19	104.04	0.42
2MS6	0.2443561	0.0026	0.005	276.92	136.15	0.30
2SM6	0.2471781	0.0050	0.006	26.31	75.48	0.74
3MK7	0.2833149	0.0028	0.004	354.06	88.39	0.38
M8	0.3220456	0.0025	0.005	330.24	136.71	0.24

The number of form F determined from table 1 is the ratio / of amplitudes:

$F = (K1+O1)/(M2+S2)$  which makes it possible to evaluate the relative importance of the diurnal components compared to the semi-diurnal components and thus to determine the type of tide. These values are as follows: semi-diurnal:  $0 < F < 0.25$ ; **mixed**  $0.25 < F < 3$ ; and diurnal:  $3 < F$ . In the case of San Pedro,  $F = (0.104+0.0403)/(0.3488+0.1303) = 0.301$

What confirms the results of *Mahan and ABE* [15] which showed the mixed nature of the tide in San Pedro.

### III-5. Propagation of the tidal wave

Relative phases of each harmonic component of the tide compared to spindle GMT allow inter alia estimating the travel time of the tide between the quay constraint (harbor office) and the fishing port.

The description of the water levels showed that the semi-diurnal component is the component of tide which is most important for both stations. In the two stations, the component m2 shares its prevalence with other components (S2 and K1) but it is always most dominant with a rough ratio report/ signal of 20 at station 1 and 17 at station 2 (*Tables 2 and 3*).

The characteristics of the propagation of the component m2 are as follows:

#### III-5-1. Travel time

The travel time of the tidal wave between the two stations is estimated by the relative phases of the wave m2:

Time (hours) = (phase1-phase2) \* (period of M2/360)

With phase1 = phase of m2 at station 1

phase2 = phase of m2 at station 2

The tidal wave m2 is a semi-diurnal principal lunar wave of period 12.42 hours.

#### Numerical application:

$T = (166.39 - 157.76) \times 12.42/360 = 0.31257 \text{ H}$

**T = 18 mn 45 s**

The wave M2 to go from the quay of constraint to the quay of the fishing port is 18mn45s. . In other words, when the tide is high at station 1, it will last 18 mn 45 s to be also high at station 2, e.g. the water level in the quay of the fishing port observes a time shift difference of 18 mn 45 s compared to that of the quay of constraint.

### **III-5-2. Propagation velocity**

The estimated distance between the two tide gauges is 975 m for a time of courses of 18 mn 45. The horizontal rate of travel of the tidal wave is of:

$$V = d/t$$

$$V = 975 \text{ m}/1125.252 \text{ s}$$

$$V = \mathbf{0.8665 \text{ m/s}}$$

The wave M2 moves with a speed of the order of 0.8665 m/s in the roads of the San Pedro port. This low speed of tide will produce only weak tide currents and thus water in the port will be very quiet and the ships moored within the port will undergo only weak currents of tides.

## **IV – CONCLUSION**

The tidal wave in the port of San Pedro is propagated with a speed of about 0.9 m/s between the quay of constraint and the quay of the fishing port from approximately 1 km. The laps resulting from the tidal wave have a very weak influence on the harbor structures as well as the ships moored in the port. The water levels in the two stations are in dipphase with a shift of approximately 19 minutes. The average height of tide in the port is approximately 1.2 m.

The fall and the rise of the water level can be well explained only if a correlation is made between the time variations of atmospheric pressure, wind and marine current compared to the time series of water level.

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