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ADAPTIVE HYBRID FORCE-POSITION CONTROL OF A ROBOTIC MANIPULATOR

Donatien NGANGA-KOUYA

Mechanical engineering department, Ecole Normale Supérieure de l'Enseignement Technique (ENSET)
B. P. 3989 Libreville,Gabon

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* Correspondance et tirés à part, e-mail : *ngad1109@yahoo.com*

ABSTRACT

A new hybrid force-position control method for uncertain robotic manipulator interacting with its environment is presented. First, the dynamical model system in the compliance frame is derived from the usual joint frame model and leads to two sets of equations due to the constraint associated to the contact surface. Next, the two dynamics are separately used for the synthesis of position and force tracking controllers. For the position control part, the design method consists of an estimated-parameters dependent on coordinate transformation and a control law derived from a backstepping procedure. The force control law has two folds: first it compensates the dynamical interaction between the end-effector motion and the force induced by the environment and secondly, imposes a desired force using a proportional-like equation. Finally, a parameter-adaptation algorithm derives from stability criteria and dependent both on the position and force tracking errors. Simulation results on a four-degree of freedom robotic system tracking a triangle while maintaining a constant contact force prove the effectiveness of our solution.

Keywords : *Control, bakstepping, force, position, adaptive.*

RÉSUMÉ

Contrôle adaptatif de commande hybride force-position pour un manipulateur robotique

Une nouvelle méthode de commande hybride force-position pour un manipulateur robotique incertain, en interaction avec son environnement est présentée. Dans un premier temps, le modèle dynamique du système dans le référentiel de l'outil est obtenu à partir du modèle dynamique usuel dans le référentiel des joints qui donne deux ensembles d'équations des contraints associées à la surface de contact. Ensuite, ces ensembles d'équations de contraintes en deux dynamiques sont utilisées séparément pour la synthèse des contrôleurs de force et position. Pour la commande de position, la méthode de conception consiste à faire une estimation des paramètres qui dépendent de la transformation des coordonnées et de la loi de commande découlant de la procédure du backstepping. La loi de commande de force a deux objectifs : premièrement, Il compense les interactions dynamiques entre le mouvement de fin de l'effecteur et la force induite par l'environnement et deuxièmement, il impose une force désirée en utilisant une structure semblable au contrôleur proportionnel. Finalement, un algorithme d'adaptation des paramètres est obtenu à partir du critère de stabilité and des erreurs de suivie de position et de force. Les résultats de simulation sur un système robotique à quatre degrés de liberté faisant un suivi de trajectoire triangulaire, tout en maintenant une force de contact constante, prouvent l'efficacité de la méthode proposée.

Mots-clés : *Commande, backstepping, force, position, adaptatif*

I - INTRODUCTION

Numerous robotic tasks (e.g. contour following, grinding, scribing, deburring and assembly-related tasks) generate physical contact between the robot end-effector and the environment. In these cases, the force due to the contact with the surface has to be taken into consideration and therefore both the force and motion control are required. Two goals are related to this problem. The first is to maintain a certain force magnitude applied by the end-effector on the surface and the second is to maintain the end-effector's motion on a desired trajectory. This problem has been intensively studied in the last two decades and two major approaches emerge. The impedance control approach proposed by Hogan [1] aims to control the position and the force by adjusting the mechanical impedance of the end-effector to external forces generated by the contact with the environment. This class of solution can further be divided into dynamic and classical impedance control when the manipulator dynamics are taken into account or not. The second approach proposed by [2] is referred to hybrid control. The directions in which the manipulator end-effector position should be controlled and the ones in which the contact force control is performed are selected so as to simultaneously follow a given desired trajectory and force. Again, taking arm dynamics into account or not, to increasing stability and performance leads to two subclasses. The works of Khatib [3], Yoshikawa [4] and McClamroch and al. [5] are majors contributions in the hybrid force –position approach. The hybrid control synthesis can be directly performed either in the joint space [4] or the task (Cartesian) space where the specifications are naturally given [7]. The actuator dynamics may [8] or may not be incorporated to the system to be controlled. Several interesting alternative solutions for the position-force control problems are also reported in the literature [6] and references therein. In addition, there exist some interactions between robot and motor dynamics which cannot be neglected. Hence in this paper, we consider a robotic system consisting of a manipulator arm and its joints actuators. A novel adaptive hybrid force position technique which guarantees asymptotic convergence of both the arm and motor variables is proposed. It consists of first, deriving the dynamical model system in the compliance frame from the usual joint frame model and lead to two sets of equations due to the constraint associated to contact surface. Next, the two dynamics are separately used for the synthesis of position and force tracking controllers. For the position control part, the design method consists of estimated-parameters dependent coordinate transformation and control law derived using a backstepping procedure. The force control law has two folds: first it compensates the dynamical interaction between the end-effector motion and the force induced by the environment and secondly, imposes a desired force using a proportional-like equation. Finally, a parameter adaptation algorithm is derived from a stability condition and is dependent both on the position and force tracking errors. In [10], the problem of force/position tracking for a robotic manipulator in compliant contact with a surface under non-parametric uncertainties is considered. A novel neuro-adaptive controller is proposed, that exploits the approximation capabilities of the linear in the weights neural networks [10], guaranteeing the uniform ultimate boundedness of force and position. In [11] the problem of output feedback tracking control of a class of Euler-Lagrange systems subject to nonlinear dissipative loads is approached. The proposed controller-observer scheme renders the origin of the error dynamics uniformly globally asymptotically stable

This paper is organized as follows. The manipulator and actuators dynamical models in the compliance frame are derived in the first part of section 2. Under the assumption that the system parameters are perfectly known, a design method so that the end-effector tracks a desired position while it maintains a desired force with the environment is presented in the second portion of section 3. The adaptive method is given in the third part of section 3. It is assumed that only the manipulator parameters are unknown. Numerical simulation results are offered in section 3. Finally, a conclusion is drawn in section 4.

II - FUNDAMENTAL CONCEPTS

II-1. Robot Dynamics

II-1-1. Manipulator Dynamics

An n degrees of freedom rigid robot arm with environmental contact can be described as

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + g(q) = \tau_e + \tau_f \quad (1)$$

with $D(q) \in R^{n\times n}$ is the symmetric and positive definite joint space inertia matrix, $B(q,\dot{q})\dot{q} \in R^{n\times 1}$ represents the Coriolis and centrifugal terms, $g(q) \in R^{n\times 1}$ represents the gravitational terms, $q \in R^{n\times 1}$ is the joint angles matrix, $\tau_e \in R^{n\times 1}$ and $\tau_f \in R^{n\times 1}$ are the motor (actuators) torques and the interaction torques due to contact with the environment respectively.
$B(q,\dot{q})$ can be written such that $\dot{D}(q) - 2B(q,\dot{q})$ is skew-symmetric. The robot can also be written in linear form in terms of a parameters vector θ as follows:

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + g(q) = Y(q,\dot{q},\ddot{q})\theta \quad (2)$$

where Y is a matrix of known functions and $\theta \in R^r$ is the vector of constant parameters.
The interaction torque $\tau_f \in R^{n\times 1}$ in the joint space is related to the interaction force $f \in R^{m\times 1}$ (m is the dimension of the task space which is assumed to be the same as the Cartesian space) at the end-effector through

$$\tau_f = J^T(q) f \quad (3)$$

where $J(q) \in R^{m\times n}$ is the manipulator task Jacobian matrix that is supposed to be nonsingular.

II-1-2. Actuators Dynamics

The main objective of robot force-position control is to compute the required torque for the robot to follow a desired trajectory of position and force. DC motors are usually used to generate the driving robot arms torques. The actuators dynamics can be described by

$$L_j \frac{di_j}{dt} + R_j i_j + K_j^e \omega_j = u_j \qquad j = 1,2,\cdots,n \tag{4}$$

with ω_j being the robot angular velocity, R_j the armature circuit resistance, L_j the armature circuit inductance, K_j^e the motor back EMF constant, i_j and u_j the armature current and voltage respectively.

The robot angular velocity ω_j is related to the joint velocity $\dot{q}_j$ through

$$\omega_j = N_j \dot{q}_j \qquad j = 1,2,\cdots,n \tag{5}$$

where N_j is the *j*th joint gear ratio. The armature current and the motor torque are related by

$$\tau_{ej} = N_j K_j^T i_j \qquad j = 1,2,\cdots,n \tag{6}$$

where K_j^T is the *j*th motor torque constant. Expressing the armature current i_j in terms of the motor torque τ_j and substituting the resulting expression and (5) into (4) yields

$$\frac{L_j}{N_j K_j^T} \frac{d\tau_{ej}}{dt} + \frac{R_j}{N_j K_j^T} \tau_{ej} + K_j^e N_j \frac{dq_j}{dt} = u_j \qquad j = 1,2,\cdots,n \tag{7}$$

The equation (7) can be written in a closed matrix form as

$$M\dot{\tau}_e + N\tau_e + K_m \dot{q} = u \tag{8}$$

Where $M = diag\{ L_j / (N_j K_j^T) \}$, $N = diag\{ R_j / (N_j K_j^T) \}$ $K_m = diag\{ K_j^e N_j \}$ $j = 1,2,\cdots,n$.

Equations (1) and (8) completely describe both the constrained manipulator interacting with the environment and its actuators in the joint frame. Since the nature description for the end effector trajectories and interaction forces during constrained motion is given in a coordinate frame fixed in the task-oriented space, it appears convenient to use a description of the manipulator and actuators dynamics in that frame.

II-1-3. Manipulator Dynamics in the Cartesian Frame

For convenience the robot and actuator dynamics are rewritten below

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + g(q) = \tau_e + \tau_f \tag{9a}$$

$$M\dot{\tau}_e + N\tau_e + K_m \dot{q} = u \tag{9b}$$

The end-effector position and orientation in the Cartesian space is related to the joint angles by $r = h(q)$ and $\dot{r} = J(q)\dot{q}$

where $r \in R^{m\times 1}$ is the position and orientation vector and $h(q)$ is the robot kinematic transformation function.

The end-effector motion equations can then be written as

$$H(r)\ddot{r} + C(r,\dot{r})\dot{r} + G(r) = f_e + f \qquad (10a)$$

$$M_r \dot{f}_e + N_r f_e + K_r \dot{r} = u_r \qquad (10b)$$

where the relationship between the joint space and task space quantities is established through the following equations:

$H(r) = J^{-T}(q)D(q)J^{-1}(q)$, $C(r,\dot{r}) = J^{-T}(q)B(q,q)J^{-1}(q) - H(r)\dot{J}(q)J^{-1}(q)$

$G(r) = J^{-T}(q)g(q)$,

$f_e = J^{-T}(q)\tau_e$, $M_r = J^{-T}(q)MJ^{T}(q)$, $N_r = J^{-T}(q)NJ^{T}(q) + M_r J^{-T}(q)\dot{J}^{T}(q)$

$K_r = J^{-T}(q)K_m J^{-1}(q)$ and $u_r = J^{-T}(q)u$

II-1-4. Robot Dynamics in the Compliance Frame

Let ${}^0S$ be the base reference of Cartesian frame. The end-effector orientation r is usually described in ${}^0S$ in terms of roll-pitch-yaw or any proper set of Euler angles. However, the rotational part of a robotic task is described in terms of angular velocity and torques in ${}^0S$. If x denotes the vector of linear and angular velocities then the following general relation hold

$$\dot{x}_c = T(r)\dot{r} \qquad (11)$$

The orientation-dependent matrix $T(r)$ is invertible, except for isolated points and means that the angular is not an exact differential of the orientation.

Let ${}^tS$ be a frame associated to the task and usually referred as compliance frame. The desired force f_d and trajectory x_d and velocity $\dot{x}_d$ associated to the task are naturally expressed in this frame. Linear velocity and force are transformed from ${}^0S$ into ${}^tS$ by means of a 3×3 rotation matrix $R_L(r)$. Similarly, angular velocity and torque are transformed by $R_A(r)$. Therefore, the following relation holds

$$\dot{x}_c = R(r)\dot{x} = R(r)T(r)\dot{r} \qquad (12)$$

Where $R(r) = \begin{bmatrix} R_L(r) & 0 \\ 0 & R_A(r) \end{bmatrix}$

with $R^{-1}(r) = R^{T}(r)$,

and $\dot{x}_c \in R^{6\times1}$ denotes the linear and angular velocity vector describes into ${}^tS$.

For simplicity let $L(r) = T^{-1}(r)R^{-1}(r)$ then (12) becomes

$$\dot{r} = L(r)\dot{x}_c \qquad (13)$$

The end-effector motion equation in the compliance frame can be easily obtained:

$$H_c(r)\ddot{x}_c + C_c(r,\dot{r})\dot{x}_c + G_c(r) = F_{ce} + F_{cint} \quad (14a)$$

$$M_c\dot{f}_e + N_c f_e + K_c\dot{x}_c = u_c \quad (14b)$$

where

$F_{cint} = L^T f$, $H_c = L^T H(r)L$, $C_c = L^T C(r)L + L^T H(r)\dot{L}$, $G_c = L^T(q)G(r)$, $F_{ce} = L^T f_e$,
$M_c = M_r$ $N_c = N_r$, $K_c = K_r L$, $u_c = u_r$

The velocity vector $\dot{x}_c$ can be partitioned according to the division of constrained and unconstrained directions

$$\dot{x}_c = \begin{pmatrix}\dot{x}_p & \dot{x}_f\end{pmatrix}^T \quad (15)$$

Since we assume that both the tool and the contact surface are rigid, the end-effector motion in the constrained direction is negligible compared to the motion in the unconstrained one. Therefore, the velocity vector $\dot{x}_c$ can be written as

$$\dot{x}_c = \begin{pmatrix}\dot{x}_p & 0\end{pmatrix}^T \quad (16a)$$

The interaction force also has the following decomposition, since its components in the unconstrained directions are negligible.

$$F_{cint} = \begin{pmatrix}0 & f_c\end{pmatrix}^T \quad (16b)$$

with f_c denoting the contact force in the force-controlled directions.
The constrained motion also induces a certain decomposition of the joint velocity vector $\dot{q}$ and the relation (16c) below holds.

$$\dot{q} = \begin{pmatrix}\dot{q}_c & 0\end{pmatrix}^T \quad (16c)$$

Since $\dot{x}_c$ and $\dot{q}$ are related through the Jacobian matrix $J(q)$ the latter must have the following structure

$$J = \begin{pmatrix} J_{c1} & J_{c2} \\ 0 & J_{c3} \end{pmatrix} \quad (17)$$

Introducing (16a, 16b) and (17) into (14a and 14b) gives the end-effector motion and the actuators dynamics both in the constrained and unconstrained directions,

$$H_{c11}\ddot{x}_p + C_{c11}\dot{x}_p + G_{c1} = F_1 \quad (18a)$$

$$M_{c11}\dot{F}_1 + N_{c11}F_1 + K_{c11}\dot{x}_p = u_{c1} \quad (18b)$$

$$H_{c21}\ddot{x}_p + C_{c21}\dot{x}_p + G_{c2} = F_2 + f_c \quad (19a)$$

$$M_{c21}\dot{F}_2 + N_{c21}F_2 + K_{c21}\dot{x}_p = u_{c2} \quad (19b)$$

where the following notation has been used

$$F_{ce} = (F_1 \quad F_2)^T \qquad G_c = (G_{c1} \quad G_{c2})^T$$

$$X_c = \begin{bmatrix} X_{c11} & X_{c12} \\ X_{c21} & X_{c22} \end{bmatrix}, \quad X_c \in \{H_c, C_c\}, \quad Z_c = \begin{bmatrix} Z_{c11} & 0 \\ 0 & Z_{c21} \end{bmatrix}, \quad X_c \in \{M_c, N_c, K_c\}$$

It is important to point out that equations (18a, 19a) have been already obtained and used in [7]. Introducing the actuators stage in the manipulator dynamics leads to the decoupled equations (18b, 19b). The particular form of the Z_c matrix is induced by the forms of the matrices M , N , K_m and J .
The following sections are devoted to the synthesis of u_{c1} and u_{c2} so that the end-effector tracks a desired trajectory x_d while maintaining a desired constant force f_d with the environment. The dynamics (18a and 18b) are used for the position tracking problem while (19a and 19b) for the force control. First, the model parameters are assumed to be known and a non-adaptive version of the controller is derived. The coordinate transformation and control laws involved in the non-adaptive version are transposed in the adaptive one with the unknown parameters replaced by their estimation. The parameters estimation algorithm is derived from stability criteria.

II-2. Controller Design

In this section, the manipulator and the actuators dynamics equations parameters are assumed to be known. The design objective is to satisfy the specifications described below.

II-2-1. Design Specifications

Determine u_{c1} and u_{c2} so that the position $x_p(t)$ follows the desired trajectory $x_{pd}(t)$ and the applied force $f_c(t)$ follows the desired force $f_d(t)$ as $t \rightarrow \infty$.

II-2-2. Position Control

Let us consider the dynamics (18a and 18b) and define

$$\left. \begin{aligned} x_1 &= x_p \\ x_2 &= \dot{x}_p \\ x_3 &= F_1 \\ v_1 &= u_{c1} \end{aligned} \right\} \qquad (20)$$

Its state space representation is given by

$$\dot{x}_1 = x_2 \qquad (21a)$$

$$H_{c11}\dot{x}_2 = -C_{c11}x_2 - G_{c1} + x_3 \qquad (21b)$$

$$M_{c11}\dot{x}_3 = -N_{c11}x_3 - K_{c11}x_2 + v_1 \qquad (21c)$$

The tracking error dynamics can be described as follows

$$\dot{e}_1 = e_2 \tag{22a}$$

$$H_{c11}\dot{e}_2 = -C_{c11}e_2 - G_{c1} + e_3 + \{-C_{c11}x_{2d} + x_{3d} - H_{c11}\dot{x}_{2d}\} \tag{22b}$$

$$M_{c11}\dot{e}_3 = -N_{c11}e_3 - K_{c11}e_2 + v_1 + \{-N_{c11}x_{3d} - K_{c11}x_{2d} - M_{c11}\dot{x}_{3d}\} \tag{22c}$$

with the error variables defined by

$$e_1 = x_1 - x_{1d} \qquad e_2 = x_2 - x_{2d} \qquad e_3 = x_3 - x_{3d}$$

Where

$$x_{1d} = x_{pd} \qquad x_{2d} = \dot{x}_{pd} \qquad x_{3d} = F_{1d}$$

The variable F_{1d} stands for the desired value of F_1.

Proposition 1:

Using a Backstepping procedure, the system described by equations (22a, 22b and 22c) is asymptotically stable if the control input v_1 is such that

$$v_1 = N_{c11}e_3 + K_{c11}e_2 + M_{c11}\dot{x}_{3d} + N_{c11}x_{3d} + K_{c11}x_{2d} + M_{c11}\dot{e}_3^* + M_{c11}H_{c11}\{-K_3\xi_3 - \xi_2\} + M_{c11}\dot{H}_{c11}\xi_3 \tag{23}$$

where

$$\xi_2 = e_2 + K_1e_1$$

$$H_{c11}\xi_3 = e_3 - C_{c11}e_2 - G_{c1} - C_{c11}x_{2d} + x_{3d} - H_{c11}\dot{x}_{2d} - H_{c11}(-K_2\xi_2 - \xi_1)$$

$$e_3^* = C_{c11}e_2 + G_{c1} + C_{c11}x_{2d} - x_{3d} \quad H_{c11}\dot{x}_{2d} - H_{c11}K_1e_2 + H_{c11}(-K_2\xi_2 - \xi_1)$$

II-2-3. Force Control

Let now consider the dynamics (19a and 19b) and set $v_2 = u_{c2}$. Differentiating (19b) and replacing $\dot{x}_p$ and $\ddot{x}_p$ by their expressions from respectively (19b) and (19a) yields the following equivalent dynamics of the set (19a and 19b),

$$\alpha\dot{v}_2 + \beta v_2 = \delta\dot{F}_2 + \gamma\ddot{F}_2 + \eta F_2 - G_{c2} + f_c \tag{24}$$

where

$\alpha = H_{c21}K_{c21}^{-1}$;

$\beta = (C_{c21}K_{c21}^{-1} - H_{c21}K_{c21}^{-1}\dot{K}_{c21}K_{c21}^{-1})$;

$\delta = \{H_{c21}K_{c21}^{-1}N_{c21} + C_{c21}K_{c21}^{-1}M_{c21} + H_{c21}K_{c21}^{-1}\dot{M}_{c21} - H_{c21}K_{c21}^{-1}\dot{K}_{c21}K_{c21}^{-1}M_{c21}\}$;

$\gamma = H_{c21}K_{c21}^{-1}M_{c21}$;

$\eta = C_{c21}K_{c21}^{-1}N_{c21} + I - H_{c21}K_{c21}^{-1}\dot{K}_{c21}K_{c21}^{-1}N_{c21} + H_{c21}K_{c21}^{-1}\dot{N}_{c21}$

The input v_2 can be decomposed in 2 components. The first v_{21} has the objective to compensate the dynamical interaction between the end-effector motion and the force induced by the environment. The second component v_{22}, as explained later, can be obtained by a simple proportional control law. Therefore

$$v_2 = v_{21} + v_{22} \tag{25}$$

We select v_{21} such that

$$\begin{cases} \dot{z} = \alpha^{-1}(-\beta z + \delta \dot{F}_2 + \gamma \ddot{F}_2 + \eta F_2 - G_{c2} + f_d) \\ v_{21} = z \end{cases} \tag{26}$$

It is important to note that even if equation (26) involves derivatives of some variables, the output v_{21} is differentiable (hence continuous) and is one of the advantages of taking into account the actuator dynamics during the design process.

The resulting dynamics after replacing (25 and 26) into (24) is

$$\alpha \dot{v}_{22} + \beta v_{22} + f_d = f_c \tag{27}$$

We propose to choose v_{22} of the form

$$v_{22} = -K_p(f_c - f_d) = -K_p \Delta f \tag{28}$$

which is indeed a simple proportional control law. K_p is a positive definite matrix. The closed loop force dynamics are given by

$$\alpha K_p \Delta \dot{f} + (\beta K_p + I)\Delta f = 0 \tag{29}$$

If K_p is selected such that

$$\beta K_p + I > 0, \tag{30}$$

then, the system described by (24) is asymptotically stable; hence the tracking force error converges to zero.

Remark 1

A particular value of K_p satisfying (30) can be computed after finding $\beta_{max} = \sup_{\pi}(\beta)$, such that π is the robot workspace. Then $K_p > -\beta_{max}^{-1}$.

II-2-4. Adaptive Controller Design

In this section, we assume that all robot parameters are unknown except those of the actuators.

II-2-4-1. Position Control

The nonlinear coordinate transformation is time-varying since it depends on estimated parameters.

$$\hat{\xi}_1 = e_1 \tag{31a}$$

$$\hat{\xi}_2 = e_2 + K_1 e_1 \tag{31b}$$

$$\hat{H}_{c11}\hat{\xi}_3 = e_3 - \hat{C}_{c11}e_2 - \hat{G}_{c1} - \hat{H}_{c11}\dot{x}_{2d} + x_{3d}$$
$$-\hat{C}_{c11}x_{2d} + \hat{H}_{c11}\{ K_2 e_2 + K_1 e_2 + K_2 K_1 e_1 + e_1 \} \tag{31c}$$

where $\hat{X} \in \{ \hat{H}, \hat{C}, \hat{M}, \hat{N}, \hat{K} \}$ is the estimated version of the unknown parameters dependent function $X \in \{ H, C, M, N, K \}$. The control input expression is chosen to be of the form (similar to (23) but with estimated parameters).

$$v_1 = \hat{N}_{c11}e_3 + \hat{K}_{c11}e_2 + \hat{N}_{c11}x_{3d} + \hat{K}_{c11}x_{2d} +$$
$$\hat{M}_{c11}\dot{x}_{3d} + \hat{M}_{c11}\hat{H}_{c11}\{ -K_3\hat{\xi}_3 - \hat{\xi}_2 \} + \hat{M}_{c11}\hat{e}_3^* + \hat{M}_{c11}\dot{\hat{H}}_{c11}\hat{\xi}_3 \tag{32}$$

Differentiating (31a, 31b and 31c) one obtains

$$\dot{\hat{\xi}}_1 = -K_1\hat{\xi}_1 + \hat{\xi}_2 \tag{33a}$$

$$\dot{\hat{\xi}}_2 = -K_2\hat{\xi}_2 - \hat{\xi}_1 + \hat{\xi}_3 - \hat{H}_{c11}^{-1}\{ \Delta H_{c11}\dot{e}_2 +$$
$$\Delta C_{c11}e_2 + \Delta G_{c1} + \Delta H_{c11}\dot{x}_{2d} + \Delta C_{c11}x_{2d} \}$$
$$= -K_2\hat{\xi}_2 - \hat{\xi}_1 + \hat{\xi}_3 - \hat{H}_{c11}^{-1}W_2\Delta p \tag{33b}$$

$$\dot{\hat{\xi}}_3 = -K_3\hat{\xi}_3 - \hat{\xi}_2 - \hat{H}_{c11}^{-1}W_3\Delta p - \hat{H}_{c11}^{-1}\Psi_3\Delta\dot{p} \tag{33c}$$

Where (since the linearly parametrizability property holds)

$$W_2\Delta p = \{ \Delta H_{c11}\dot{e}_2 + \Delta C_{c11}e_2 + \Delta G_{c11} + \Delta H_{c11}\dot{x}_{2d} + \Delta C_{c11}x_{2d} \}$$
$$W_3\Delta p = \{ \Delta N_{c11}e_3 + \Delta K_{c11}e_2 + \Delta N_{c11}x_{3d} + \Delta K_{c11}x_{2d} +$$
$$\Delta M_{c11}\dot{x}_{3d} + \Delta M_{c11}\dot{e}_3 \}$$
$$\Psi_3\Delta p = \dot{\hat{C}}_{c11}e_2 + \dot{\hat{G}}_{c1} + \dot{\hat{C}}_{c11}x_{2d} + \dot{\hat{H}}_{c11}\dot{x}_{2d}$$
$$-\dot{\hat{H}}_{c11}(K_2 e_2 + K_1 e_2 + K_2 K_1 e_1 + e_1)$$
$$\Delta X = X - \hat{X}\ .$$

The parameter $\Delta p = p - \hat{p}$ denotes the difference between the unknown robot parameters vector p and their estimation $\hat{p}$ from an adaptive algorithm yet to be determined. Note that if $\Delta X = 0$ then $\Delta p = 0$.

II-2-4-2. Force Control

Again, the control input v_2 decomposition given by equation (25) is adopted and, v_{21} and v_{22} are respectively selected such that

$$\begin{cases} \dot{z} = \hat{\alpha}^{-1}(-\hat{\beta}z + \hat{\delta}\dot{F}_2 + \hat{\gamma}\ddot{F}_2 + \hat{\eta}F_2 - \hat{G}_{c2} + f_d) \\ v_{21} = z \end{cases} \tag{34}$$

$$v_{22} = -K_p(f_c - f_d) = -K_p\Delta f \tag{35}$$

The closed loop force dynamics is then described by

$$\hat{\alpha} K_p \Delta \dot{f} = -(\hat{\beta} K_p + I)\Delta f + W_4 \Delta p \qquad (36)$$

where the matrix K_p is such that

$$s(K_p) = \hat{\alpha}^{-1}\hat{\beta} K_p + \hat{\alpha}^{-1}$$

is positive definite. The lengthy expression of W_4 is omitted for the sake of simplicity. Since $s(K_p)$ involves estimated parameters, offline computation of K_p requires prior knowledge of estimated parameters range or bounds. If this is not the case, the designer can force the estimated parameters to remain into a predefined domain using a projection adaptation algorithm as mentioned in remark 1.

II-2-4-3. Parameters Adaptation algorithm

Rearranging equations (33) and (36) in a closed form yields,

$$\begin{pmatrix} \dot{\hat{\xi}} \\ \dot{\hat{\xi}}_2 \\ \dot{\hat{\xi}}_3 \\ \Delta \dot{f} \end{pmatrix} = \begin{bmatrix} -K_1 & I & 0 & 0 \\ -I & -K_2 & I & 0 \\ 0 & -I & -K_3 & 0 \\ 0 & 0 & 0 & -s(K_p) \end{bmatrix} \begin{pmatrix} \hat{\xi}_1 \\ \hat{\xi}_2 \\ \hat{\xi}_3 \\ \Delta f \end{pmatrix} + \begin{pmatrix} 0 \\ -\hat{H}_{c1}^{-1} W_2 \\ -\hat{H}_{c1}^{-1} W_3 \\ \alpha^{-1} W_4 \end{pmatrix} \Delta p + \begin{pmatrix} 0 \\ 0 \\ -\hat{H}_{c1}^{-1} \Psi_3 \\ 0 \end{pmatrix} \Delta \dot{p} \qquad (37)$$

or equivalently $\dot{\hat{\xi}} = A_s \hat{\xi} + W \Delta p + \Psi \Delta \dot{p}$

where $\hat{\xi} = \begin{pmatrix} \hat{\xi}_1 & \hat{\xi}_1 & \hat{\xi}_1 & \Delta f \end{pmatrix}^T$. Note that A_s is Hurwitz since the gains K_i, $i = 1, \cdots, 3$ and $s(K_p)$ are positive definite.

We now introduce the augmented error y and the error augmentation $\hat{\varepsilon}$ defined by

$$y = \hat{\xi} - \hat{\varepsilon} \qquad (38)$$

$$\dot{\hat{\varepsilon}} = A_s \hat{\varepsilon} + \Psi \Delta \dot{p} \quad \hat{\varepsilon}(0) = 0 \qquad (39)$$

The augmented error dynamics are then,

$$\dot{y} = A_s y + W \Delta p \qquad (40)$$

Next we define the following positive definite Lyapunov function

$$V = \frac{1}{2} y^T y + \Delta p^T \Gamma^{-1} \Delta p \qquad (41)$$

Γ is a positive definite matrix. The derivative of V is

$$\dot{V} = y^T(A_s^T + A_s)y + 2\Delta p^T(\Gamma^{-1}\Delta p + W^T y) \qquad (42)$$

If the estimation dynamics are select such that

$$\Delta\dot{p} = -\Gamma W^T y, \ \Gamma > 0 \qquad (43)$$

The derivative of V becomes

$$\dot{V} = y^T(A_s^T + A_s)y \qquad (44)$$

Since A_s is Hurwitz and $A_s = A_s^T$ then $(A_s^T + A_s)$ is negative definite, therefore $\dot{V}$ is negative semi-definite. We can show that the estimated parameters are bounded and the position and force tracking error converge to zero.

Remark 2: projection algorithm

The projection variant given below can be used instead of (43) to force the estimated parameters to remain in a predefined domain, say, $\Omega_x = \{x : \|x\| \leq x_{max}\}$.

$$\Delta\dot{p} = -\dot{\hat{p}} = -\Gamma \, Proj(W^T y, \hat{p}) \qquad (45)$$

with the $Proj()$ function is defined as follow

$Proj(y, \hat{\theta}) = y$, if $f(\hat{\theta}) \leq 0$

$Proj(y, \hat{\theta}) = y$, if $f(\hat{\theta}) \geq 0$ and $\frac{\partial f}{\partial \hat{\theta}} y \leq 0$

$$Proj(y, \hat{\theta}) = \left[I - f(\hat{\theta})(\frac{\partial f}{\partial \hat{\theta}})^T(\frac{\partial f}{\partial \hat{\theta}}) \Big/ \left\| \frac{\partial f}{\partial \hat{\theta}} \right\|^2 \right] y,$$

if

$$f(\hat{\theta}) > 0 \text{ and } \frac{\partial f}{\partial \hat{\theta}} y > 0$$

where $f(\hat{\theta}) = (\|\hat{\theta}\|^2 - \hat{\theta}_{max}) / (\varepsilon + 2\varepsilon\hat{\theta}_{max})$ and ε is a positive real number. The function $f(\hat{\theta})$ is usually called the boundary function. If $\hat{\theta}(t)$ is inside the compact set or on its boundary and attempts to remain inside it, the adaptation law is equivalent to the gradient method. The algorithm forces each estimated parameter to remain into Ω_x by subtracting a suitable value to y when $\hat{\theta}(t)$ is on the boundary and has the tendency to move away from it. Indeed, the term

$$(I - f(\hat{\theta})(\frac{\partial f}{\partial \hat{\theta}})^T(\frac{\partial f}{\partial \hat{\theta}}) \Big/ \left\| \frac{\partial f}{\partial \hat{\theta}} \right\|^2) \qquad (46)$$

is referred to as the projected direction onto the tangent plane to the boundary.

Proposé 3

III - RESULTS AND DISCUSSION

The proposed control scheme has been tested by simulation using the robot given in figure 1. It is a four-degree of freedom robot arm with one prismatic axis, and three rotating axes. However, the motion around the z_4 axis has been locked for the test. The system's dynamic model is very big and has been then voluntarily omitted from the paper. The manipulator links mass, length and inertia, and the actuators parameters are collected into ***Table 1***. The end effector has to follow, at constant linear velocity $v_u = 0.25m/s$, a triangle (A,B,C) in a plan P describes by the equation

$$2x + y + z + 1 = 0 \tag{47}$$

Table 1 : *Robot parameters*

Weight (kg)	$m_1 = 3.75$, $m_2 = 2.5$, $m_3 = 9.165$, $m_4 = 1.151$
Length (m)	$l_1 = 0.1731$, $l_2 = 0.11229$, $l_3 = 0.1769$
Center of mass (m)	$x_2 = 0.1151$, $y_3 = 0.2301$,
Inertia (kg/dm^3)	$I_{z2} = 3$, $I_{z3} = 4$, $I_{z3} = 9.165$, $I_{x4} = 7$
L(mH)	$L_j = 3.16 \quad j = 1,\cdots 4$
R(Ω)	$R_j = 11.5 \quad j = 1,\cdots 4$
K($N.m/A$)	$K_j = 2.124 \quad j = 1,\cdots 4$
K^e($V/(rad/s)$)	$K_j^e = 2 \quad j = 1,\cdots 4$

Table 2 : *Some model parameters definition*

$p_1 = m_1 + m_2 + m_3 + m_4$
$p_2 = I_{zz2} + m_2 x_2^2 + m_4 L_2^2 + m_4 L_3^2 + m_4 z_4^2 + m_3 y_3^2 + 2 m_4 z_4 L_3$ and $p_3 = m_3 y_3^2$

Figure 2 shows a very good path tracking performance. The end effector follows the triangle with a very good accuracy. ***Figure 3*** illustrates the end effector actual and desired positions into the Cartesian coordinates. One can easily see that actual end effector positions are very close their desired values. The tracking errors given at ***Figure 4*** confirms aforementioned

analysis. The errors are negligible since they are less than 0.005. The estimated parameters are shown at Figure 5. One can note that they are bounded (the meaning of p_1, p_2 and p_3 are given in ***Table 2***). The convergence is very fast. It is worth mentioning that estimated parameters are controller parameters and they do not converge to the true robot parameters, in general. The force time-behavior is illustrated at ***Figure 6.*** We can also note that a good response is obtained thanks to the relative simple but effective force control equations (46 and 47). The adaptive feature considerably helps reducing the control efforts. The actuator input voltages and currents are shown in ***Figures 6*** and ***7***. These signals are bounded too and they remain in acceptable ranges.

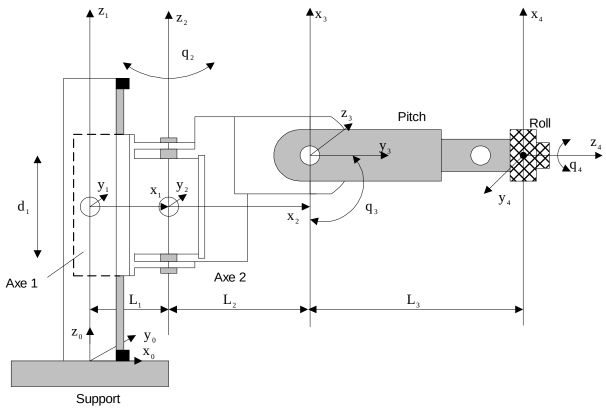

Figure 1 : *The four-DOF Robotic Manipulator*

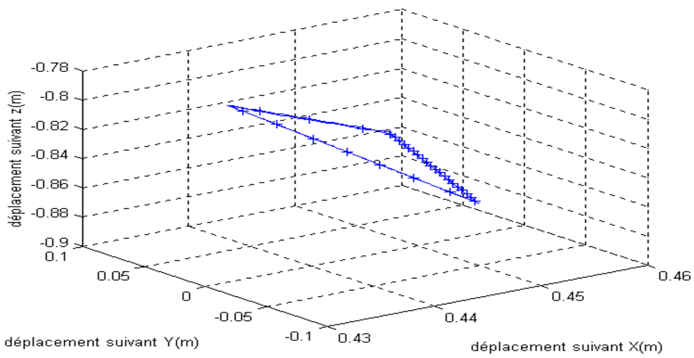

Figure 2 : *3-dimensional representation of the position tracking performance*

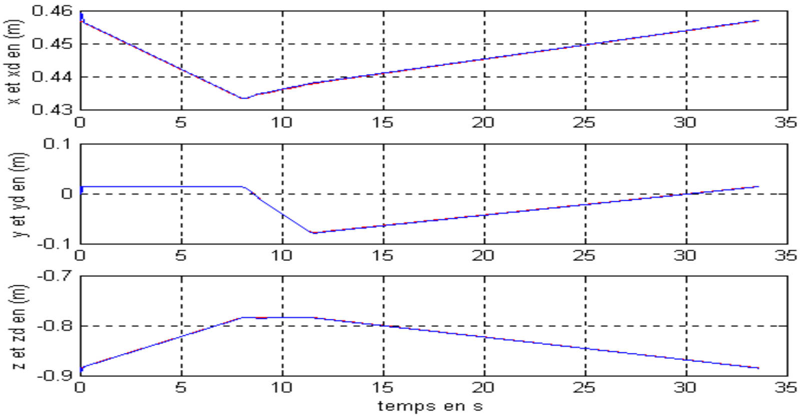

Figure 3 : *End effector actual and desired positions*

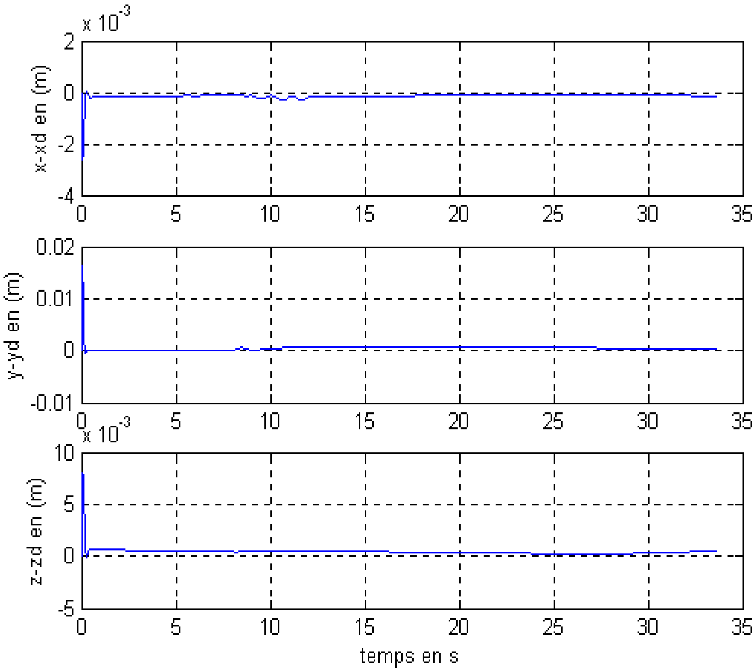

Figure 4 : *Position tracking error waveforms*

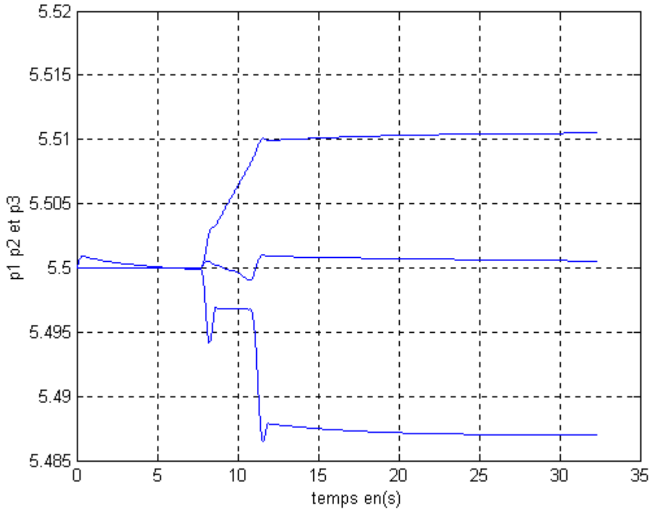

Figure 5 : *Some estimated parameters*

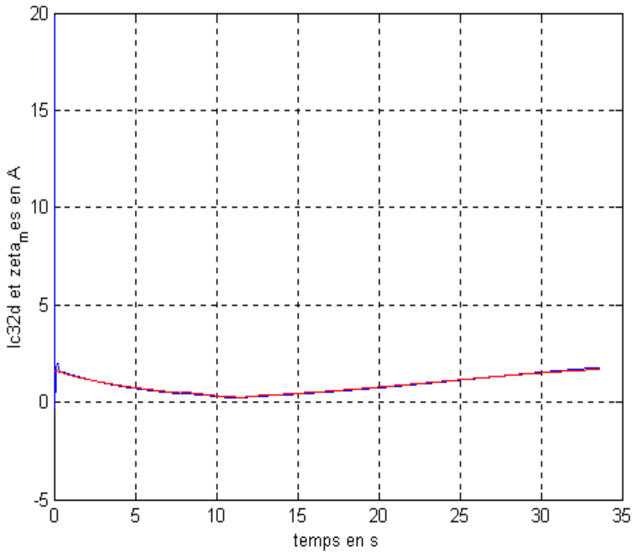

Figure 6 : *Actual and desired actuators the currents profiles*

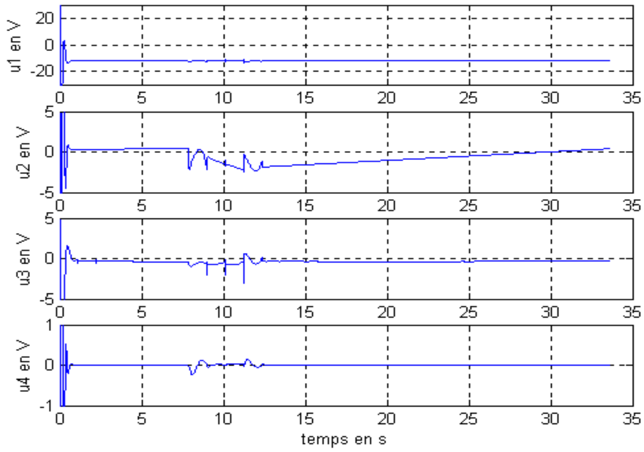

Figure 7 : *Joint 1, 2, 3 and 4 actuators voltage profile*

IV - CONCLUSION

A new adaptive force-position control method is proposed for a manipulator taking into account actuators dynamics and uncertainties on robot's parameters. The robot is in contact with a rigid environment. Simulations were performed with a four-link manipulator arm and the results prove the effectiveness of our method since very good path and force tracking performance are achieved. Although the mathematical development used throughout the design is relatively involved, the control laws derived are relatively simple to implement. This implementation will be the next step of this work

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